

# ABSTRACT BOOK

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# CONTENTS

<b>CONTENTS</b>	<b>i</b>
<b>1. FOREWORD</b>	<b>xiv</b>
<b>2. COMMITTEES</b>	<b>xv</b>
<b>3. SESSIONS</b>	<b>xviii</b>
<b>4. ACKNOWLEDGMENTS</b>	<b>xix</b>
<b>5. PLENARY SPEAKERS</b>	<b>1</b>
$\sigma$ - Convergence and a New Sequence Space of Order $\alpha$ <i>Ekrem Savas</i>	<b>2</b>
Mathematical Beauty in Black Hole Radiation <i>İzzet Sakallı</i>	<b>3</b>
Simulation: It's the Name of the Game <i>Jane Horgan, Charlie Daly, James Power</i>	<b>4</b>
Variations of the ccc Property <i>Ljubiša D.R. Kočinac</i>	<b>5</b>
Exponential Stability for the Nonlinear Schrödinger Equation with Locally Distributed Damping <i>Marcelo M. Cavalcanti, Wellington J. Corrêa, Turker Özsarı, Mauricio Sepúlveda, Rodrigo Véjar Asem</i>	<b>6</b>
Spectral Disjointness and Invariant Subspaces <i>Robin Harte</i>	<b>7</b>
<b>6. ABSTRACTS</b>	<b>8</b>
<b>6.1 Topology</b>	<b>9</b>
Neutrosophic Soft $\delta$ -Topology and Neutrosophic Soft Compactness <i>Ahu Acikgoz, Ferhat Esenbel</i>	<b>10</b>
Neutrosophic Soft Pre-Separation Axioms <i>Ahu Acikgoz, Ferhat Esenbel</i>	<b>11</b>
Neutrosophic Soft Semiregularization Topologies and Neutrosophic Soft Submaximal Spaces <i>Ahu Acikgoz, Ferhat Esenbel</i>	<b>12</b>
Domination in Discrete Topology Graphs <i>Ali Ameer Jabor, Ahmed abd-Ali Omran</i>	<b>13</b>

<b>On Intuitionistic Fuzzy Cone Symmetric Spaces</b>	<b>14</b>
<i>Banu Pazar Varol</i>	
<b>On A Uniform Analogue of Paracompact Spaces</b>	<b>15</b>
<i>Bekbolot Kanetov, Anara Baidzhuranova</i>	
<b>Some Properties of Remainders of Uniform Spaces and Uniformly Continuous Mappings</b>	<b>16</b>
<i>Bekbolot Kanetov, Ulukbek Saktanov, Dinara Kanetova</i>	
<b>On Some Properties of Completeness of Uniform Spaces</b>	<b>17</b>
<i>Bekbolot Kanetov, Dinara Kanetova, Meerim Zhanakunova</i>	
<b>Notes on Free Topological Groups</b>	<b>18</b>
<i>Chuan Liu</i>	
<b>On Mean Intuitionistic Open Sets</b>	<b>19</b>
<i>Esra Dalan Yildirim</i>	
<b>Some Generalizations of Caristi Type Fixed Point Theorem on <math>M</math>-Metric Spaces</b>	<b>20</b>
<i>Hakan Sahin, Ishak Altun, Duran Turkoglu</i>	
<b>Topological Aspect of Monodromy Groupoid for a Topological Internal Groupoid</b>	<b>21</b>
<i>Hurmet Fulya Akız, Osman Mucuk</i>	
<b>On Strong Pre-Continuity with Fuzzy Soft Sets</b>	<b>22</b>
<i>Huseyin Cakalli, Ahu Acikgoz, Ferhat Esenbel</i>	
<b>Direct Sum of Barreled Locally Convex Cones</b>	<b>23</b>
<i>Mohammad Reza Motallebi</i>	
<b>Crossed Module Aspects of Monodromy Groupoids for Internal Groupoids</b>	<b>24</b>
<i>Osman Mucuk, Serap Demir, Tunçar Şahan</i>	
<b>A Condition for Points and Compact Subsets of <math>C(X)</math> to be <math>G_\delta</math> Subsets of <math>\mathbb{R}^X</math></b>	<b>25</b>
<i>Smail Kelaiaia</i>	
<b>G– Fuzzy Sequential Continuity in FTS</b>	<b>26</b>
<i>Taja Yaying, Ahu Acikgoz, Huseyin Cakalli</i>	
<b><math>m^*</math>-<math>g</math>-Closed Sets in Minimal Spaces <math>(X, m)</math> with Hereditary Hlasses due to Császár</b>	<b>27</b>
<i>Takashi Noiri, Ahu Acikgoz</i>	
<b>The Shortest Length Distance and the Digital <math>r</math>-Thickening on Digital Images</b>	<b>28</b>
<i>Tane Vergili</i>	
<b>On Measures of Parameterized Fuzzy Compactness</b>	<b>29</b>
<i>Vildan Çetkin</i>	
<b>Chain Connectedness</b>	<b>30</b>
<i>Zoran Misajleski, Nikita Shekutkovski, Emin Durmishi</i>	
<b>6.2 Analysis and Functional Analysis</b>	<b>31</b>
<b>Reduced Approximation for Operator with Time in Parabolic Problems Case</b>	<b>32</b>
<i>Ali Berkane, Mohamed Belhout</i>	

<b>Stabilization of the Wave Equation with a Localized Memory Term and Border Friction Dissipation</b>	<b>33</b>
<i>Aries Mohammed Es-Salih</i>	
<b>Statistical Quasi Cauchy Sequences in Abstract Metric Spaces</b>	<b>34</b>
<i>Ayse Sonmez and Huseyin Cakalli</i>	
<b>Existence Results of First-Order Differential Systems with Mixed Quasimonotone Nonlinearities and Integral Boundary Condition</b>	<b>35</b>
<i>Bachir Messirdi, Mohammed Derhab, Tewfik Khedim</i>	
<b>Steklov Eigenvalue Problem with <math>\alpha</math>-Harmonic Solutions</b>	<b>36</b>
<i>Belhadj Karim</i>	
<b>Second Order Sweeping Process with Almost Convex Perturbation</b>	<b>37</b>
<i>Doria Affane, Mustapha Fateh Yarou</i>	
<b>Singular Degenerate Normal Differential Operators for First-Order</b>	<b>38</b>
<i>Fatih Yilmaz, Meltem Sertbaş</i>	
<b>Existence of Solutions for Delay Dynamic Equations on Time Scales</b>	<b>39</b>
<i>Faycal Bouchelaghem, Abdelouaheb Ardjouni, Ahcene Djoudi</i>	
<b>Optimization of Second Order Evolution Differential Inclusions Problem with Phase Constraints</b>	<b>40</b>
<i>Gulseren Cicek, Elimhan Mahmudov</i>	
<b>Global Behavior of some Difference Equations</b>	<b>41</b>
<i>Hamdy A. El-Metwally, M. AL-kaff</i>	
<b>Norm and Almost Everywhere Convergence of Convolution Powers</b>	<b>42</b>
<i>Heybetkulu Mustafayev</i>	
<b>An Extreme Point Theorem</b>	<b>43</b>
<i>Hülya Duru</i>	
<b>Nonlocal Elliptics Problems with Hardy Potential Term</b>	<b>44</b>
<i>Kheireddine Biroud</i>	
<b>Some Estimates in Homogeneous Function Spaces</b>	<b>45</b>
<i>Madani Moussai</i>	
<b>Weighted Stepanov Pseudo Almost Periodic Functions on Time Scales</b>	<b>46</b>
<i>Mohamed Zitane, Mohssine Es-saiydy</i>	
<b>On a Study of New Class of <math>q</math>-Fractional Operator</b>	<b>47</b>
<i>Mohammad Momenzadeh</i>	
<b>An Inequality for Self Reciprocal Polynomials</b>	<b>48</b>
<i>Mohammed A. Qazi</i>	
<b>On Property <math>(UW_E)</math> under Functional Calculus</b>	<b>49</b>
<i>Mohammed Kachad</i>	
<b>Pseudo Almost Periodic Generalized Functions</b>	<b>50</b>
<i>Mohammed Taha Khalladi</i>	

<b>Global Existence and Uniqueness of Solutions to a Parabolic Haptotaxis Model</b>	<b>51</b>
<i>Naima Aissa, Hocine Tsmada</i>	
<b>On the <math>\Delta_g</math>-Statistical Convergence of the Function Defined Time Scale</b>	<b>52</b>
<i>Nihan Turan, Metin Başarır</i>	
<b>A Minimization Algorithm for Limit Extremal Problems on Convex Compactum</b>	<b>53</b>
<i>Özkan Değer</i>	
<b>First Order Maximally Dissipative Singular Differential Operators</b>	<b>54</b>
<i>Pembe Ipek Al, Zameddin I. Ismailov</i>	
<b>Schatten-von Neumann Characteristic of Tensor Product Operators</b>	<b>55</b>
<i>Pembe Ipek Al, Zameddin I. Ismailov</i>	
<b>Some Properties of Solutions to Dynamical Systems</b>	<b>56</b>
<i>Serkan Ilter</i>	
<b>Sufficient Conditions of Optimality for Free Time Optimization of Third Order Differential Inclusions</b>	<b>57</b>
<i>Sevilay Demir Sağlam, Elimhan N. Mahmudov</i>	
<b>Shechter Spectra and Relatively Demicompact Linear Relations</b>	<b>58</b>
<i>Slim Fakhfakh</i>	
<b>Rate of Convergence by Phillips Operators Involving Appell Polynomials</b>	<b>59</b>
<i>Şule Yüksel Güngör, Nurhayat İspir</i>	
<b>On the Rigidity Part of Schwarz Lemma at the Boundary</b>	<b>60</b>
<i>Tuğba Akyel, Bülent Nafi Örnek</i>	
<b>The Transmission Problem for the Laplace Operator on a Domain with a Cuspidal Point</b>	<b>61</b>
<i>Wided Chikouche</i>	
<b>6.3 Sequences, Series, Summability</b>	<b>62</b>
<b>On a Summation by the Abel Method of a System of Root Functions for a Nonlocal Problem with Integral Conditions</b>	<b>63</b>
<i>Abdelhak Berkane</i>	
<b>On Rough Convergence of Triple Sequences</b>	<b>64</b>
<i>Ayhan Esi, Nagarajan Subramanian, M. Kemal Ozdemir</i>	
<b>Triple Sequence Spaces of Intuitionistic Rough <math>I</math>-Convergence Defined by Compact Bernstein Operator</b>	<b>65</b>
<i>Ayhan Esi, Nagarajan Subramanian, M. Kemal Ozdemir</i>	
<b>Necessary and Sufficient Tauberian Conditions Under Which Convergence Follows from <math>A^{r,\delta}</math> Summability</b>	<b>66</b>
<i>Çagla Kambak, Ibrahim Canak</i>	
<b>Characterization of the Compact Operators on the Class <math>(bv, bv_k^\theta)</math></b>	<b>67</b>
<i>Fadime Gökçe, M. Ali Sarıgöl</i>	
<b>Certain Matrix Characterizations Related to the Difference Spaces</b>	<b>68</b>
<i>Fadime Gökçe</i>	

<b>On Some Properties of a New Paranormed Space <math> C_{-1} (p)</math></b>	<b>69</b>
<i>G. Canan Hazar Güleç</i>	
<b>Matrix operators Involving the Space <math>bv_k^\theta</math></b>	<b>70</b>
<i>G. Canan Hazar Güleç, M. Ali Sarıgöl</i>	
<b>Some Tauberian Theorems for <math>(C, 1, 1)</math> Summability Method by Regularly Generated Integrals</b>	<b>71</b>
<i>Gökşen Fındık, İbrahim Çanak</i>	
<b>Logarithmic Summability of Integrals on <math>[1, \infty)</math></b>	<b>72</b>
<i>Gökşen Fındık, İbrahim Çanak</i>	
<b>Lacunary <math>A</math>- Statistical Convergence and Lacunary Strong <math>A</math>- Convergence of Order <math>(\alpha, \beta)</math> with Respect to a Modulus</b>	<b>73</b>
<i>Hacer Şengül, Mikail Et, Hüseyin Çakallı</i>	
<b>Deferred Statistical Convergence of Order <math>\alpha</math> in Topological Groups</b>	<b>74</b>
<i>Hacer Şengül, Mikail Et, Hüseyin Çakallı</i>	
<b>Lacunary <math>d</math>-Statistical Convergence and Lacunary <math>d</math>-Statistical Boundedness in Metric Spaces</b>	<b>75</b>
<i>Hacer Şengül, Mikail Et, Hüseyin Çakallı</i>	
<b>Rho Statistical Convergence of Order Beta</b>	<b>76</b>
<i>Huseyin Cakalli, Hacer Sengul Kandemir</i>	
<b>Abel Statistical Convergence in Metric Spaces</b>	<b>77</b>
<i>Huseyin Cakalli</i>	
<b>Tauberian Theorems for the Weighted Mean Summability of Integrals on <math>[1, \infty)</math></b>	<b>78</b>
<i>Ibrahim Canak, Firat Ozsarac</i>	
<b>Category Theoretical View of I-Cluster and I-Limit Points for Ideals <math>I</math> with the Baire Property</b>	<b>79</b>
<i>Leila Miller-Van Wieren, Tugba Yurdakadim, Emre Tas</i>	
<b>Deferred Statistical Convergence and Strongly Deferred Summable Functions</b>	<b>80</b>
<i>Mikail Et, P. Baliarsingh, Hacer Şengül</i>	
<b>Lacunary Statistical Convergence of Difference Sequences of Fractional Order</b>	<b>81</b>
<i>Nazlım Deniz Aral, Mikail Et</i>	
<b>Application on Local Properties of Factored Fourier Series</b>	<b>82</b>
<i>Şebnem Yıldız</i>	
<b>A New Generalization on Absolute Riesz Summability</b>	<b>83</b>
<i>Şebnem Yıldız</i>	
<b>Absolute Matrix Summability on Quasi Power Increasing Sequences</b>	<b>84</b>
<i>Şebnem Yıldız</i>	
<b>Sufficient Tauberian Conditions for the <math>(\bar{N}, p)</math> Summability of Sequences</b>	<b>85</b>
<i>Sefa Anıl Sezer, İbrahim Çanak</i>	
<b><math>p</math>-Ward Continuity in 2-Normed Spaces</b>	<b>86</b>
<i>Sibel Ersan</i>	

<b>6.4 Fixed Point Theory</b>	<b>87</b>
<b>New Approach to Find Multi-Fractal Dimension of Multi- Fuzzy Fractal Attractor Sets Based on Iterated Function System</b>	<b>88</b>
<i>Arkan Jassim Mohammed</i>	
<b>On the Study of Nonlinear Fractional Differential Equations on Unbounded Interval</b>	<b>89</b>
<i>Boulares Hamid</i>	
<b>Chatterjea Type Fixed Point Results on Metric Spaces</b>	<b>90</b>
<i>Gonca Durmaz Gungor, Ishak Altun, Ali Cihad Kabaci</i>	
<b>Fixed Point Results with <math>\theta</math>-Function on Quasi Metric Space</b>	<b>91</b>
<i>Gonca Durmaz Gungor, Ishak Altun</i>	
<b>Common Fixed Points for Two Mappings</b>	<b>92</b>
<i>Hakima Bouhadjera</i>	
<b>Complexity Analysis of Primal-Dual Interior Point Methods for Semidefinite Programming Based on a New Kernel Function with an Hyperbolic Barrier Term</b>	<b>93</b>
<i>Imene Touil, Wided Chikouche</i>	
<b>Almost Picard Operators</b>	<b>94</b>
<i>Ishak Altun, Hatice Aslan Hancer</i>	
<b>Nonconvex Second Order Sweeping Processes in Hilbert Spaces</b>	<b>95</b>
<i>Lounis Sabrina, Haddad Tahar</i>	
<b>A Logarithmic Barrier Method for Linear Programming using a New Minorant Function</b>	<b>96</b>
<i>Menniche Linda, Benterki Djamel, Merikhi Bachir</i>	
<b>Existence and Uniqueness of Solution for the Problem in the Times Scales</b>	<b>97</b>
<i>Mouhamed Nehari</i>	
<b>Fixed Point Approach for Differential Inclusions Governed by Subdifferential Operators</b>	<b>98</b>
<i>Mustapha Fateh Yarou, Doria Affane</i>	
<b>Some Fixed Point Results on Modular <math>\mathcal{F}</math>-Metric Spaces</b>	<b>99</b>
<i>Nesrin Manav, Duran Turkoglu</i>	
<b>Three-Step Projective Methods for Solving the Split Feasibility Problems</b>	<b>100</b>
<i>Nontawat Eiamniran, Nattawut Pholasa, Prasit Cholamjiak</i>	
<b>A Fixed Point Approach for a Differential Inclusion Governed by the Subdifferential of PLN Functions</b>	<b>101</b>
<i>Nora Fetouci, Mustapha Fateh Yarou</i>	
<b>Fixed Point Theorems on Orthogonal Metric Spaces via Altering Distance Functions</b>	<b>102</b>
<i>Nurcan Bilgili Gungor, Duran Turkoglu</i>	
<b>Fixed Point Theory on Spaces with Vector-Valued Metrics</b>	<b>103</b>
<i>Safia Bazine</i>	
<b>Homoclinic and Heteroclinic Bifurcations for Henon Like Diffeomorphism</b>	<b>104</b>
<i>Selmani Wissame, Djellit Ithem</i>	

<b>Convergence Theorems for Three G-Nonexpansive Mappings in Hilbert Spaces with Graphs by Modifying SP and Noor Iterations with Shrinking Projection Methods</b>	<b>105</b>
<i>Supitcha Pheetarakorn, Nattawut Pholasa, Watcharaporn Chalamjiak</i>	
<b>Existence Results of First-Order Differential Equations without Integral Boundary Conditions at Resonance</b>	<b>106</b>
<i>Tewfik Khedim, Mohammed Derhab, Bachir Messirdi</i>	
<b>6.5 Numerical Functional Analysis</b>	<b>107</b>
<b>Solvability of a Mixed Problem for a Heat Equation with an Involution Perturbation</b>	<b>108</b>
<i>Abdisalam Sarsenbi</i>	
<b>Mixed Problem for a Wave Equation with an Involution Perturbation</b>	<b>109</b>
<i>Abdizhahan Sarsenbi, Madina Utelbaeva</i>	
<b>Numerical Analysis of the Influence of the Initial Parameters on the Convergence Rate of the Approximate Solution of the Boundary Value Problem</b>	<b>110</b>
<i>Akylbek Kerimbekov, Aijana Ermekbaeva, Gulnaz Mombekova</i>	
<b>A Crank Nicolson Difference Scheme for the System of Nonlinear Observing Epidemic Models</b>	<b>111</b>
<i>A. Ashyralyev, E. Hincal, B. Kaymakamzade</i>	
<b>A Numerical Algorithm for the Source Identification Parabolic-Elliptic Problem</b>	<b>112</b>
<i>Allaberen Ashyralyev, Cagin Arıkan</i>	
<b>A Numerical Algorithm for the Source Identification Elliptic-Telegraph Problem</b>	<b>113</b>
<i>Allaberen Ashyralyev, Ahmad Al-Hammouri</i>	
<b>A Numerical Algorithm for the Involutory Parabolic Problem</b>	<b>114</b>
<i>Allaberen Ashyralyev, Amer Mohammed Saeed Ahmed</i>	
<b>Finite Difference Method for the Third-Order Partial Differential Equation with Nonlocal Boundary Conditions</b>	<b>115</b>
<i>Allaberen Ashyralyev, Kheireddine Belakroum</i>	
<b>Time-Dependent Source Identification Problem for the Schrödinger Equation with Nonlocal Boundary Conditions</b>	<b>116</b>
<i>Allaberen Ashyralyev, Mesut Urun</i>	
<b>A Numerical Algorithm for the Third Order Partial Differential Equation with Time Delay</b>	<b>117</b>
<i>A. Ashyralyev, E. Hincal, S. Ibrahim</i>	
<b>A Numerical Algorithm for the Involutory Schrödinger Type Problem</b>	<b>118</b>
<i>A. Ashyralyev, Twana Abbas</i>	
<b>Numerical Solution of the System of Fractional Differential Equations Observing Epidemic Models</b>	<b>119</b>
<i>Allaberen Ashyralyev, B. Kaymakamzade, L.D. Hayder</i>	
<b>Well-Posedness of Source Identification Problem for the Elliptic Equation in a Banach Space</b>	<b>120</b>
<i>Allaberen Ashyralyev, Charyyar Ashyralyyev</i>	



<b>On Elliptic Differential and Difference Problems in a Hilbert Space with Special Type Non-local Conditions</b>	<b>121</b>
<i>Allaberen Ashyralyev, Ayman Hamad</i>	
<b>Optimal Control Approach to Study Two Strain Malaria Model</b>	<b>122</b>
<i>Bashir Abdullahi Baba, Parvaneh Esmaili, Isa Abdullahi Baba</i>	
<b>On a Boundary Problem for a Nonlocal Poisson Equation with Boundary Operators of the Hadamard Type</b>	<b>123</b>
<i>Batirkhan Turmetov, Rakhim Shamsiev</i>	
<b>On the Numerical Analysis of a Nonlinear Fractional Schrödinger Equation with Neumann Boundary Condition</b>	<b>124</b>
<i>Betul Hicdurmaz</i>	
<b>An Iterative Regularization Method for an Ill-Posed Bi-Fractional Problem</b>	<b>125</b>
<i>Boussetila Nadjib, Sassane Roumaissa, Rebbani Faouzia</i>	
<b>Numerical Solution of Neumann Type Elliptic Overdetermined Multipoint Mixed Boundary Value Problem</b>	<b>126</b>
<i>Charyyar Ashyralyev, Suzan Karabey</i>	
<b>A New Coupling Method for the Stokes-Darcy Model</b>	<b>127</b>
<i>C.Ziti, R.Malek</i>	
<b>Sinc Approximation of Solution of Integro-Differential Equation</b>	<b>128</b>
<i>Dounia Belakroum, Kheireddine Belakroum</i>	
<b>Differential and Difference Variants of 2-d Nonlocal Boundary Value Problem with Poisson's Operator</b>	<b>129</b>
<i>Dovlet M. Dovletov</i>	
<b>Posterior Analysis of Weighted Erlang Distribution</b>	<b>130</b>
<i>E. Hincal, Sultan Alsaadi</i>	
<b>Memory Efficient Algorithm for Solving the Inverse Gravimetry Problem of Finding Several Boundary Surfaces in Multilayered Medium</b>	<b>131</b>
<i>Elena N. Akimova, Vladimir E. Misilov, Murat A. Sultanov, Rauan Zh. Turebekov</i>	
<b>Numerical Analysis of Convergence Rate of Approximation Solutions of a Boundary Value Problem for Oscillation Processes</b>	<b>132</b>
<i>Elmira Abdylldaeva, Zarina Kabaeva, Kubat Karabakirov</i>	
<b>Solving Nonlinear Volterra-Fredholm Integro-Differential Equations Using He's Variational Iteration Method</b>	<b>133</b>
<i>Fernane Khaireddine, Ellagoune Selma</i>	
<b>The Effect of Harvesting Policy on an Eco-Epidemiological model</b>	<b>134</b>
<i>Karrar Q. AL-Jubourili, Reem M. Hussien, Nadia M.G. Alsaïdi</i>	
<b>Interpolation of Scattered Data in <math>\mathbb{R}^3</math> Using Minimum <math>L_p</math>-Norm Networks, <math>1 &lt; p &lt; \infty</math></b>	<b>135</b>
<i>Krassimira Vlachkova</i>	
<b>On Unique Solvability of Nonlinear Coupled Systems</b>	<b>136</b>
<i>Meltem Uzun, Ozgur Yildirim</i>	

<b>Construction of Unconditionally Stable Difference Schemes Based on Stability of Perturbed Difference Scheme</b>	<b>137</b>
<i>Murat A. Sultanov, Musabek I. Akylbaev</i>	
<b>An Inverse Diffusion-Wave Problem Defined in Heterogeneous Medium with Additional Boundary Measurement</b>	<b>138</b>
<i>Nouiri Brahim, Djerioui Khayra</i>	
<b>Symmetry Analysis of the Discrete Nonlinear Boundary Value Problems for the Wave Equation</b>	<b>139</b>
<i>Sumeyra Caglak, Ozgur Yildirim</i>	
<b>6.6 Computer Science and Technology</b>	<b>140</b>
<b>A New Pseudo Random Bits Generator via 2D chaotic System, Diffusion, and Permeation</b>	<b>141</b>
<i>Alaa kadhim Farhan, Abeer Tariq, Rasha S.Ali, Nadia M.G. Alsaidi</i>	
<b>Diffuse Representation of Image and Its Applications to Cryptography and Compression</b>	<b>142</b>
<i>Amrane Houas, Zouhir Mokhtari</i>	
<b>Comparison of C, Java, Ruby and Matlab Programming Languages Using Fibonacci Algorithm</b>	<b>143</b>
<i>Ceren Cubukcu, Zeynep Behrin Guven Aydin, Ruya Samli</i>	
<b>Image encryption based on highly sensitive chaotic system</b>	<b>144</b>
<i>Dalia S. Ali, Nawras A. Alwan, Nadia M.G. Alsaidi</i>	
<b>Review of the Most Popular Data Science Programs Used Today: Python and R</b>	<b>145</b>
<i>Ender Sahinaslan</i>	
<b>A Study on the Industrial Usage Areas of Modern Information Technologies</b>	<b>146</b>
<i>Ender Sahinaslan, Onder Sahinaslan</i>	
<b>Market Basket Analysis Developed On The Refrigerator Control System With Machine Learning Algorithms In Data Mining Applications</b>	<b>147</b>
<i>Engin Oguzay</i>	
<b>Sentiment Analysis of Turkish Twitter Data</b>	<b>148</b>
<i>Harisu Abdullahi Shehu, Sezai Tokat, Md. Haidar Sharif, Sahin Uyaver</i>	
<b>Motion Estimation from Noisy Image Sequences using New Frequency Weighting Functions</b>	<b>149</b>
<i>Ismaili Alaoui El Mehdi</i>	
<b>A Modification of Gravitational Search Algorithm with Hyper-Ellipsoids</b>	<b>150</b>
<i>İclal Gör, Korhan Günel</i>	
<b>Controlling Electrical Appliance by Thinking in Mind</b>	<b>151</b>
<i>Md. Haidar Sharif, Sahin Uyaver</i>	
<b>Development of an Information System for Storing Digitized Works of the Almaty Akademgorodok Research Institutes</b>	<b>152</b>
<i>Nurlan Temirbekov, Dossan Baigereyev, Almas Temirbekov, Bakytzhan Omirzhanova</i>	
<b>Encryption Algorithms in Blockchain Technology</b>	<b>153</b>
<i>Onder Sahinaslan</i>	

<b>Darboux integrability and algebraic invariants of an enzymatic diffusion-reaction system</b>	<b>154</b>
<i>Orhan Ozgur Aybar</i>	
<b>Weak stabilization of a fractional output for a class of semi-linear Dynamical Systems</b>	<b>155</b>
<i>R. Larhrissi, H. Zitane, A. Boutoulout</i>	
<b>Fusion of Finger-Knuckle-Print and Finger Vein Recognition using Random Forest Tree</b>	<b>156</b>
<i>Rachid Chelaoua, Abdallah Meraoumia</i>	
<b>Extracting a New Fractal and Semi-Variance Attributes for Texture Images Categorization</b>	<b>157</b>
<i>Suhad A. Yousif, Hussam Y. Abdul-Wahed, Nadia M.G. Al-Saidi</i>	
<b>A Block-Based Image Encryption Scheme Using Cellular Automata With Authentication Capability</b>	<b>158</b>
<i>Ziba Eslami, Saeideh Kabirirad</i>	
<b>6.7 Mathematical Methods in Physics</b>	<b>159</b>
<b>Two Dimensional Free Surface Flows Past an Obstacle</b>	<b>160</b>
<i>Abdelkader Gasmı</i>	
<b>Analytical Solution for the Conformable Fractional Telegraph Equation by Fourier Method</b>	<b>161</b>
<i>Abdelkebir Saad, Nowiri Brahim</i>	
<b>Small Divisors in the Solar System</b>	<b>162</b>
<i>Angel Zhivkov</i>	
<b>Laguerre Polynomial Approach for Solving Functional Differential Equations Involving First Order Nonlinear Delay Terms</b>	<b>163</b>
<i>Burcu Gürbüz, Mehmet Sezer</i>	
<b>Laguerre Matrix-Collocation Technique to Solve Systems of Functional Differential Equations with Variable Delays</b>	<b>164</b>
<i>Burcu Gürbüz</i>	
<b>Secure Optical Communication Based on New 2D-Hyperchaotic Map</b>	<b>165</b>
<i>Dhurgham Younus, Nadia M.G. Alsaıdı, Walıd K.Hamoudı</i>	
<b>The Explicit Relation Between the DKP Equation and the Klein-Gordon Equation</b>	<b>166</b>
<i>Djahida Bouchefra, Badredine Boudjedaa</i>	
<b>Soliton Solutions of Gurseı Model with Bichromatic Force</b>	<b>167</b>
<i>Eren Tosyalı, Fatma Aydođmuş</i>	
<b>MHD Micropolar Blood Flow Model through a Multiple Stenosed Artery</b>	<b>168</b>
<i>Esam A. Alnussairı, Ahmed Bakheet</i>	
<b>Total Reduction of Chiral Oscillator and Its Dirac Analysis</b>	<b>169</b>
<i>Filiz Çađatay Uçgun</i>	
<b>Thin-Shell Wormhole in <math>f(R)</math> Gravity</b>	<b>170</b>
<i>S. Habib Mazharımousavi</i>	
<b>Mathematical Beauty in Black Hole Radiation</b>	<b>171</b>
<i>İzzet Sakallı</i>	

<b>Solving Advection Equation Using the Natural Decomposition Method</b>	<b>172</b>
<i>Jeerawan Saelao, Khanittha Kamdee</i>	
<b>Application of Differential Transform Method and Adomian Decomposition Method for Solving of one Nonlinear Boundary-Value-Transmission Problem</b>	<b>173</b>
<i>Merve Yücel, Oktay Mukhtarov</i>	
<b>Mathematical Behaviour of Solutions of the Kirchhoff Type Equation with Logarithmic Non-linearity</b>	<b>174</b>
<i>Nazlı Irköl, Erhan Pişkin</i>	
<b>Modeling and Analysis the Effect of Social Media for a Zika Virus Transmission with Beddington DeAngelis Incidence Rate</b>	<b>175</b>
<i>Puji Andayani</i>	
<b>Non-Commutative Geometry and Application to Schrödinger Equation with Certain Central Potentials</b>	<b>176</b>
<i>Zaiem Slimane</i>	
<b>6.8 Recent Themes on Controllability and Stability of PDE's</b>	<b>177</b>
<b>Controllability Results of Hilfer Fractional Differential Inclusions with Non-Dense Domain in Abstract Space</b>	<b>178</b>
<i>Assia Boudjerida, Djamila Seba, Karima Laoubi</i>	
<b>A Note on Dirichlet Problem for Partial Differential Equations with Complex Variables in the Bidisc</b>	<b>179</b>
<i>Bahriye Karaca</i>	
<b>On the Stabilization of Infinite Dimensional Bilinear Systems</b>	<b>180</b>
<i>El Hassan Zerrik, Abderrahman Ait Aadi</i>	
<b>A Mixed Relaxed-Singular Optimal Controls for Systems of Forward-Backward Stochastic Differential Equations of Mean-Field Type</b>	<b>181</b>
<i>Gherbal Boulakhras, Ninouh Abdelhakim</i>	
<b>Stability Result for an Abstract Delayed Evolution Equation with Arbitrary Decay in Viscoelasticity</b>	<b>182</b>
<i>Houria Chellaoua, Yamna Boukhatem</i>	
<b>Hopf-Like Bifurcation and Mixed Mode Oscillation in a Fractional-Order FitzHugh-Nagumo Model</b>	<b>183</b>
<i>Mohammed Salah Abdelouahab, René Lozi</i>	
<b>6.9 Applied Statistics</b>	<b>184</b>
<b>Regularization Schemes for Statistical Inverse Problems</b>	<b>185</b>
<i>Abhishake Rastogi</i>	
<b>Forecasting ATM Transactions</b>	<b>186</b>
<i>Ayşe Cilacı Tombuş, Erdal Albayrak</i>	
<b>Improved New Liu-Type Estimator for Poisson Regression Models</b>	<b>187</b>
<i>Esra Ertan, İsmail Müfit Giresunlu, Kadri Ulaş Akay</i>	

<b>Generalized First Passage Time Method for the Estimation of the Parameters of the Stochastic Differential Equation of the Black-Scholes Model</b>	<b>188</b>
<i>Khaldi Khaled, Meddahi Samia</i>	
<b>Comparison of ARIMA, Holt-Winters, and LSTM Forecasting Models Using Kullback Information Measure</b>	<b>189</b>
<i>Luckshay Batra, H.C.Taneja</i>	
<b>On the Fuzzy Reliability Estimation for Lomax Distribution</b>	<b>190</b>
<i>Nadia Hashim Al-Noor</i>	
<b>On the Exponentiated Weibull Distribution</b>	<b>191</b>
<i>Nadia Hashim Al-Noor, Salah Hamza Abid, Mohammad Abd Alhussein Boshi</i>	
<b>2D Markov-Switching Autoregressive (MS AR) Models for Image Segmentation</b>	<b>192</b>
<i>Soumia Kharfouchi, Houria Djafri</i>	
<b>Entropy as a Measure of Implied Volatility in Options Market</b>	<b>193</b>
<i>H.C. Taneja, Luckshay Batra</i>	
<b>Using Copulas to Model Dependence Between Crude Oil Prices</b>	<b>194</b>
<i>Vadoud Najjari</i>	
<b>6.10 Geometry and Mathematical Education</b>	<b>195</b>
<b>Perfect Fluid Pseudo Concircular Ricci Symmetric Spacetimes with Codazzi Type Z-Tensor</b>	<b>196</b>
<i>Ayse Yavuz Tasci, Fusun Ozen Zengin</i>	
<b>On Ruled Surfaces of Natural Lift Curves and Tangent Bundle of Pseudo-Sphere</b>	<b>197</b>
<i>Emel Karaca, Mustafa Çalıřkan</i>	
<b>On Infinitesimal Transformations Of Weyl Manifolds</b>	<b>198</b>
<i>İlhan Gül</i>	
<b>Minimum Distance Between two Ellipses</b>	<b>199</b>
<i>Ivaylo Tounchev</i>	
<b>Some Properties of Generalized Complex Space Forms</b>	<b>200</b>
<i>Pegah Mutlu</i>	
<b>Pointwise Slant Semi-Riemannian Submersions from Lorentzian Para-Sasakian Manifolds</b>	<b>201</b>
<i>Sahar Mashmouli, Morteza Faghfourı</i>	
<b>Geometrical View on Set-Theoretical Solutions of Yang-Baxter Equation via Lie Algebras</b>	<b>202</b>
<i>Serife Nur Bozdog, Ibrahim Senturk</i>	
<b>6.11 Algebra and Number Theory</b>	<b>203</b>
<b>On the Aggregating of Some Fuzzy Relations and their Related Structures</b>	<b>204</b>
<i>Abdelaziz Amroune, Aissa Bouad</i>	
<b>An Abstract Characterization of Menger Algebras of Strongly Quasi-Open Multiplace Maps</b>	<b>205</b>
<i>Firudin Muradov</i>	
<b>New Types of Uninorms on Bounded Lattices</b>	<b>206</b>
<i>Gül Deniz Çaylı</i>	

<b>Sheffer Stroke Reduction for Some Algebraic Structures</b>	<b>207</b>
<i>Ibrahim Senturk, Tahsin Oner</i>	
<b>Some Properties of EM Rings</b>	<b>208</b>
<i>Manal Ghanem, Emad Abu Osba</i>	
<b>On a Generalized Identity of a Prime Ring Involving <math>b</math>-Generalized Derivations</b>	<b>209</b>
<i>Nihan Baydar Yarbil</i>	
<b>On cyclic codes of length <math>8p^s</math> over <math>\mathbb{F}_{p^m} + u\mathbb{F}_{p^m}</math></b>	<b>210</b>
<i>Saroj Rani</i>	
<b>On Products of Irreducible Characters</b>	<b>211</b>
<i>Temha Erkoc, Burcu Cinarci</i>	

# 1. FOREWORD

On behalf of the Organizing Committee, we are very pleased to welcome you to the 3<sup>rd</sup> International Conference of Mathematical Sciences (ICMS 2019) to be held between 4-8 September 2019 at Maltepe University in Istanbul.

We hope that, ICMS 2019 will be one of the most beneficial scientific events, bringing together mathematicians from all over the world, and demonstrating the vital role that mathematics play in any field of science.

Welcome to our conference, Maltepe University, İstanbul!

**Hüseyin Çakallı**  
**Chairman of the Organizing Committee**

## 2. COMMITTEES

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## 3. SESSIONS

The lectures in the following parallel sessions are to be held after the plenary speakers lectures:

1. "**Topology**" organized by Ljubisa D. R. Kocinac,
2. "**Analysis and Functional Analysis**" organized by Ibrahim Canak,
3. "**Sequences, Series, Summability**" organized by Hacer Sengul,
4. "**Fixed Point Theory**" organized by Duran Turkoglu,
5. "**Numerical Functional Analysis**" organized by Allaberen Ashyralyev,
6. "**Computer Science and Technology**" organized by Sahin Uyaver,
7. "**Mathematical Methods in Physics**" organized by Özay Gürtuğ and İzzet Sakallı.
8. "**Recent themes on Controllability and Stability of PDE's**" organized by Valria Neves Domingos Cavalcanti, and Marcelo Moreira Cavalcanti,
9. "**Applied Statistics**" organized by Mujgan Tez, and Kadri Ulas Akay,
10. "**Geometry, and Mathematical Education**" organized by Ilhan Gul.
11. "**Algebra and Number Theory**" organized by Temha Erkoç Yilmazturk.

## 4. ACKNOWLEDGMENTS

We thank firstly the founder of Maltepe University, Hüseyin ŞİMŞEK, the rector of Maltepe University, Prof.Dr. Şahin KARASAR. We also thank the paralell session organizers, and then all scientific committee members who reviewed abstracts which made the conference better.

There are many people who spent a lot of time and effort to make this conference possible. We would like to thank especially to the following young colleagues who had contributed to the success of this conference in various ways:

**Önder Şahinaslan**, Maltepe University, Turkey

**Fuat Usta**, Düzce University, Istanbul, Turkey

**Özkan Değer**, Istanbul University, Istanbul, Turkey

## 5. PLENARY SPEAKERS

The abstracts of the plenary lectures are given in the following.

## $\sigma$ - Convergence and a New Sequence Space of Order $\alpha$

Ekrem Savas

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A continuous linear functional  $\phi$  on  $l_\infty$  is said to be an invariant mean or a  $\sigma$ -mean if and only if

1.  $\phi(x) \geq 0$  when the sequence  $x = (x_k)$  has  $x_n \geq 0$  for all  $n$ ;
2.  $\phi(e) = 1$  where  $e = (1, 1, 1, \dots)$  and
3.  $\phi(x_{\sigma(n)}) = \phi(x)$  for all  $x \in l_\infty$ .

In the this paper, we present a new sequence space of order  $\alpha$  which is defined by using invariant mean and infinite matrix. Also some inclusion theorems are proved.

**Keywords:** Modulus function,  $\varphi$ -function,  $\lambda$ - strong convergence, matrix transformations, sequence spaces, order  $\alpha$ .

**2010 Mathematics Subject Classification:** 40H05; 40C05,

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Mathematical Beauty in Black Hole Radiation

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Hawking considered how quantum particles behaved close to a black hole, predicting that a black hole irradiated a form of thermal radiation, later termed Hawking radiation [1]. Although the classical black holes are asymptotically flat, especially in the presence of axion and dilaton fields, which are the dark matter and energy candidates, black holes can no longer be an asymptotically flat. The best example for this type is the rotating linear dilaton black hole (RLDBH) [2, 3]. In this study, we revisit the HR radiation problem of the RLDBH [4]. For this pupose, we consider the most advanced scalar perturbations: charged massless spin-0 fields. After separating the covariant Klein-Gordon equation into radial and angular equations, the analytical solutions of those equations are obtained in terms of the confluent Heun functions. Various physical problems are discussed with the obtained analytical solutions: resonance frequencies, quantization and greybody factor [5]. Moreover, we derive the Hawking temperature of the RLDBH by using the Damour-Ruffini-Sannan method. The mathematical beauty of black hole radiation is remarkable during all these processes.

**Keywords:** Hawking Radiation, Black Hole, Dilaton, Axion, Quantization, Greybody, Heun Functions.  
**2010 Mathematics Subject Classification:** 35Q75, 37N20, 58D30, 83A05.

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## Simulation: It's the Name of the Game

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Originally, when we were giving a first course in probability to students reading for a degree in computing at Dublin City University, we used to think we were doing a good job; the students seemed happy enough. And then came the end-of-semester assessment forms: "too many formulae", "not relevant to computing", and -worst of all - "just another maths course". So we had to do something about it; we turned to  $R$  for help.

Students of computing seem to be able happily to think about Computer Architecture, Image Processing, Language Design, Software Engineering, Operating Systems and Forensics, but to freeze up when it comes to "Probability", and to wonder what it might have to do with computing.

In this paper, we outline the approach taken to convince these students of the importance of probability to computing, and along the way, to teach them a little bit about probability itself. We have largely replaced the traditional algebraic approach by simulation and experimentation, taking advantage of the powerful graphical and simulation facilities of the statistical system  $R$ , which is freely available, and downloadable, from the web.  $R$  is used not only as a tool for calculation and data analysis, but mainly to illustrate the concepts of probability, to simulate distributions, and to explore new problems. We avoid, as far as is possible mathematical details, instead encouraging students to investigate for themselves, through experimentation and simulation in  $R$ . For example, we have examined models for server queue lengths without using any of the formulae typical in a traditional approach. Recognizing that the student better understands definitions, generalizations and abstractions after seeing the applications, almost all new ideas are introduced and illustrated by real, computer-related examples, covering a wide range of computer science applications.

It is now ten years since the book "Probability with  $R$ ", outlining this approach, was published. Back then we had just begun to hear of smartphones, fitbits, apps and bluetooth; machine learning was in its infancy. It is timely to address how probability applies to new developments in computing. The applications and examples of the first edition are beginning to look somewhat passé and old fashioned. The new edition is an updated and extended version. The  $R$  code has been revamped with better coding style using  $R$  functions that have become available in recent years. The examples have been updated to reflect realistic applications of today's world of computing. New sections on packages and interfaces, machine learning, linear regression, Bayesian analysis, spam filtering and password recognition have been added.

The approach in our second edition remains the same, favouring experimentation and simulation rather than the traditional mathematical approach, integrating  $R$  programs throughout the text and encouraging students to investigate for themselves, through experimentation and simulation in  $R$ . The few algebraic derivations deemed necessary, are developed in the appendices.

Keywords: Probability, Game Theory.

2010 Mathematics Subject Classification: 03B48 , 91A80.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Variations of the ccc Property

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A topological space  $X$  has the ccc property if each collection of pairwise disjoint open subsets of  $X$  is countable. In recent years several authors considered versions of this property related to selection principles and game theory. We discuss relations among these properties, as well as their importance in selection principles theory. Some open problems will be posed.

**Keywords:** Selection principles, ccc, selectively ccc, selectively star-ccc.  
**2010 Mathematics Subject Classification:** 54D20, 91A44.

International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Exponential Stability for the Nonlinear Schrödinger Equation with Locally Distributed Damping

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This talk is concerned with the defocusing nonlinear Schrödinger equation with a locally distributed damping on a smooth bounded domain. We first construct approximate solutions for this model by using the theory of monotone operators. We show that these approximate solutions decay exponentially fast in the  $L^2$ -sense by using the multiplier technique and a unique continuation property. Then, we prove the global existence as well as the  $L^2$ -decay of solutions for the original model by passing to the limit and using a weak lower semicontinuity argument, respectively. Finally, we implement a precise and efficient algorithm for studying the exponential decay established in the first part of the paper numerically. Our simulations illustrate the efficacy of the proposed control design.

Keywords: Schrödinger equation, Exponential stability, locally distributed damping.

2010 Mathematics Subject Classification: 35Q55, 35Q93, 93D20.

## Spectral Disjointness and Invariant Subspaces

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Spectral disjointness confers a certain "independence" upon linear operators. If  $G$  is a ring with identity  $I$  then an idempotent  $Q = Q^2 \in G$  gives the ring  $G$  a block structure

$$G \cong \begin{array}{cc} A & M \\ N & B \end{array}$$

where for example  $A = QGQ$ ; then

$$T = \begin{array}{cc} a & m \\ n & b \end{array} \in G$$

commutes with  $Q$  iff it is a "block diagonal":

$$TQ = QT \iff T = \begin{array}{cc} a & 0 \\ 0 & b \end{array}.$$

Specialising to complex Banach algebras, for block diagonals there is two way implication

$$\sigma_A(a) \cap \sigma_B(b) = \emptyset \iff Q \in \text{Holo}(T) :$$

$Q = f(T)$  with  $f : U \rightarrow G$  holomorphic on an open neighbourhood of  $\sigma_G(T)$ . Weaker spectral disjointness gives a little less:

$$\sigma_A^{\text{left}}(a) \cap \sigma_B^{\text{right}}(b) = \emptyset \implies Q \in \text{comm}^2(T) :$$

the block structure idempotent  $Q$  "double commutes" with  $T \in G$ . Specializing to  $G = B(X)$ , the bounded operators on a Banach space, closed complemented subspaces  $Y \subseteq X$  give us again the block structure, and operators  $T \in G$  for which  $Y$  is "invariant" become "block triangles":

$$T(Y) \subseteq Y \iff T = \begin{array}{cc} a & m \\ 0 & b \end{array}.$$

When  $Y \subseteq X$  is not complemented then the block structure is missing and we must resort to the restriction and the quotient:

$$a = T_Y \in A = B(Y) ; b = T_{/Y} \in B(X/Y) .$$

Now spectral disjointness

$$\sigma_A(a) \cap \sigma_B(b) = \emptyset$$

ensures that the subspace  $Y \subseteq X$  is both *hyperinvariant* and *reducing*, in particular complemented.

Keywords: Keyword one, keyword two, keyword three.

2010 Mathematics Subject Classification: First, Second, Third.

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## 6. ABSTRACTS

The abstracts are ordered by the name of the first author of the presentation.

This is not a proceedings, the abstracts in this book have been printed as submitted by the authors, after revisions if requested by the referees' and the authors are responsible for the correctness of their abstracts and defending their works during their presentations at the conference.

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## 6.1 Topology

Session Organizer: Ljubiša D.R. Kočinac

There are 21 contributed presentation by 25 coauthors submitted for this session. The talks concern various topics from Topology and contain interesting results supported by well selected examples. The topics considered in the presentations are: selection principles theory, function spaces theory, free topological groups, fixed point theory in  $M$ -metric spaces, minimal structures, uniform spaces (uniform paracompactness, completeness, remainders, uniform continuity), fuzzy topological spaces, several properties of fuzzy soft topological spaces, neutrosophic soft topological spaces, intuitionistic topological spaces, intuitionistic fuzzy cone symmetric spaces, monodromy groupoids for topological internal groupoids, generalized solution of the Cauchy problem, digital topology, graph theory and topology.

We hope that these presentations will be published as full texts in mathematical journals.

International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Neutrosophic Soft $\delta$ -Topology and Neutrosophic Soft Compactness

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We introduce the concepts of neutrosophic soft  $\delta$ -interior, neutrosophic soft quasicoincidence, neutrosophic soft  $q$ -neighborhood, neutrosophic regular open soft set, neutrosophic soft  $\delta$ -cluster point, neutrosophic soft  $\delta$ -closure, neutrosophic soft  $\theta$ -cluster point, neutrosophic soft  $\theta$ -closure, neutrosophic soft  $\delta$ -neighborhood, neutrosophic semi open soft set and show that the set of all neutrosophic soft  $\delta$ -open sets is also a neutrosophic soft topology, which is called the neutrosophic soft  $\delta$ -topology. We obtain equivalent forms of neutrosophic soft  $\delta$ -continuity. Moreover, the notions of neutrosophic soft  $\delta$ -compactness and neutrosophic soft locally  $\delta$ -compactness are defined and their basic properties under neutrosophic soft -continuous mappings are investigated.

**Keywords:** Neutrosophic soft quasi-coincidence, neutrosophic regular open soft set, neutrosophic  $\delta$ -closed soft, neutrosophic semi open soft, neutrosophic soft  $\delta$ -topology.

**2010 Mathematics Subject Classification:** 54A05, 54C10, 54D30, 54D45.

## Neutrosophic Soft Pre-Separation Axioms

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In this study, we introduce the concept of neutrosophic soft pre-open (neutrosophic soft pre-closed) sets and pre-separation axioms in neutrosophic soft topological spaces. In particular, the relationship between these separation axioms are investigated. Also, we give a new definition for neutrosophic soft topological subspace and define neutrosophic soft pre-irresolute soft and neutrosophic pre-irresolute open soft functions.

**Keywords:** Neutrosophic pre-open soft set, neutrosophic soft pre-interior point, neutrosophic soft pre-cluster point, neutrosophic soft pre-separation axioms, neutrosophic soft subspace.

**2010 Mathematics Subject Classification:** 54A05, 54D10, 54D15.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Neutrosophic Soft Semiregularization Topologies and Neutrosophic Soft Submaximal Spaces

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In this study, we aim to investigate the neutrosophic soft semiregularization spaces associated with neutrosophic soft topological spaces. We introduce the concept of neutrosophic soft submaximal spaces and prove that corresponding to each neutrosophic soft topological space, there always exists a neutrosophic soft submaximal space which is an expansion of the given space. It is shown that neutrosophic soft submaximal and neutrosophic soft semiregular spaces are closely associated with those spaces which are minimal or maximal in accordance with certain types of properties which is called neutrosophic soft semiregular properties in this document. This has been an inspiration for us to deal with different characteristics for examination whether these are neutrosophic soft semiregular ones. After all, we adapt some certain mappings defined in some other types of topological spaces to neutrosophic soft topological spaces and their behaviours are investigated when either or both the domain and codomain spaces are replaced by their semiregularization spaces.

**Keywords:** Neutrosophic soft semiregularization topology, neutrosophic soft ro-equivalence, neutrosophic soft submaximal space, neutrosophic soft nearly compact space, neutrosophic soft S-closed space.

**2010 Mathematics Subject Classification:** 54A05, 54C10, 54D30, 54D10.



## Domination in Discrete Topology Graphs

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In this paper we obtain a graph from the discrete topology under some conditions taken from composition of topology, study properties of that graph and the domination number of the discrete topology graph. Finally, the affection of the discrete topology graph domination parameter when a graph is modified by deleting or adding a vertex is studied in this paper.

**Definition 1.** Let  $(X, \tau)$  be a topological space. Define the graph  $G_\tau = (V, E)$  such that:  $V = \{u : u \in \tau, u \neq \phi, X\}$   $E = \{uv \in E(G_\tau) \text{ if } u \cap v \neq \phi, u \neq v \text{ and } u, v \in \tau\}$ .

**Theorem 1.** If  $(X, \tau)$  is a discrete space and  $X$  contains greater than or equal to three elements, then  $G_\tau$  is a connected graph.

**Theorem 2.** If  $(X, \tau)$  is a discrete space with  $|X| \geq 3$ , then  $G_\tau$  has no cut vertex.

**Theorem 3.** If  $(X, \tau)$  is a discrete space with  $|X| \geq 3$ , then  $\gamma(G_\tau) = 2$ .

**Theorem 4.**  $\gamma(G_\tau - v) \leq \gamma(G_\tau)$ .

**Theorem 5.**  $\gamma(G_\tau - e) = \gamma(G_\tau)$ .

Keywords: Discrete topology, domination number

2010 Mathematics Subject Classification: 05C69, 54-XX .

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## On Intuitionistic Fuzzy Cone Symmetric Spaces

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In this work, we define intuitionistic fuzzy cone symmetric space by omitting triangle inequality in intuitionistic fuzzy cone metric space. With this structure we see some topological degenerations. We define convergence of sequences in an intuitionistic fuzzy cone symmetric space and study some sequential properties, and then give the relations between cone (sym)metric and intuitionistic fuzzy cone (sym)metric.

**Keywords:** intuitionistic fuzzy cone metric, cone symmetric, intuitionistic fuzzy cone symmetric.  
**2010 Mathematics Subject Classification** 03E72, 54E35, 03F55.

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## On A Uniform Analogue of Paracompact Spaces

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In this work we introduce and study uniformly paracompact spaces. In particular, the characterizations of uniformly paracompact spaces by using Hausdorff compact extensions and mappings are obtained.

**Definition 1.** A uniform space  $(X, U)$  is called uniformly paracompact if every finitely additive open cover of  $X$  has a  $\sigma$ -locally finite uniform refinement.

**Theorem 1.** If  $(X, U)$  is a uniform paracompact space, then the topological space  $(X, \tau_U)$  is paracompact. Conversely, if  $(X, \tau)$  is paracompact, then the uniform space  $(X, U_X)$ , where  $U_X$  is the universal uniformity, is uniformly paracompact.

**Theorem 2.** Let  $(X, U)$  be a uniform space and  $bX$  be a certain compact Hausdorff extension of  $X$ . The uniform space  $(X, U)$  is uniformly paracompact, if and only if for each compactum  $K \subset bX \setminus X$  there exists a  $\sigma$ -locally finite uniform covering  $\alpha \in U$  such that  $[A]_{bX} \cap K = \emptyset$  for all  $A \in \alpha$ .

**Theorem 3.** For a uniform space  $(X, U)$  the following are equivalent:

- (1)  $(X, U)$  is strongly uniformly paracompact;
- (2)  $(X, U)$  is uniformly paracompact and the topological space  $(X, \tau_A)$  is strongly paracompact.

Keywords: Uniform paracompactness, finitely additive open covering,  $\sigma$ -locally finite uniform covering.  
2010 Mathematics Subject Classification: 54E15, 54D20.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Some Properties of Remainders of Uniform Spaces and Uniformly Continuous Mappings

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In the theory of uniform spaces and uniformly continuous mappings one of the interesting questions is the study of remainders of uniform spaces and uniformly continuous mappings. In this work we study some properties of remainders of uniform spaces and uniformly continuous mappings. In particular, it is established the completeness,  $\tau$ -boundedness and compactness of remainders of uniform spaces, as well as the uniform perfectness of remainders of uniformly continuous mappings.

**Theorem 1.** A remainder  $(\tilde{X} \setminus X, \tilde{U}_{\tilde{X} \setminus X})$  of a uniform space  $(X, U)$  is compact if and only if the uniform space  $(X, U)$  is locally complete and each uniform covering  $\alpha \in U$  contains a finite subfamily that is a co-covering of the uniform space  $(X, U)$ .

**Theorem 2.** The remainder  $cf|_{cX \setminus X} : (cX \setminus X, cU_{cX \setminus X}) \rightarrow (Y, V)$  of the uniformly continuous mapping  $f : (X, U) \rightarrow (Y, V)$  is uniformly perfect if and only if  $X$  is open in  $(cX, cU)$ .

Keywords: co-cover, remainder, free Cauchy filter.

2010 Mathematics Subject Classification: 54E15.

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## On Some Properties of Completeness of Uniform Spaces

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One of the important concepts of uniform topology is the concept of completeness of uniform spaces. In this work we study some properties of  $\mu$ -completeness of uniform spaces. In particular, it is proved that the  $\mu$ -completeness of uniform spaces is preserved under twice uniformly continuous  $P$ -uniformly perfect mappings in both directions.

**Theorem 1.** For twice uniformly continuous and  $P$ -uniformly perfect mapping, the  $\mu$ -completeness is preserved in both the image direction and the preimage direction.

**Theorem 2.** Let  $(X, U)$  be a uniform space and  $w(X, U) \leq \tau$ . If  $(X, U)$  is  $\mu$ -complete, then  $(X, U)$  is complete.

**Theorem 3.** Let  $f : (X, U) \rightarrow (Y, V)$  be a uniformly continuous mapping of a uniform space  $(X, U)$  into a uniform space  $(Y, V)$ . If  $(Y, V)$  and  $f$  are  $\mu$ -complete, then  $(X, U)$  is also  $\mu$ -complete.

Keywords:  $\mu$ -complete, Dieudonne  $\mu$ -complete spaces, twice uniformly continuous mapping.

2010 Mathematics Subject Classification: 54E15.

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## Notes on Free Topological Groups

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In this note, we discuss some generalized metric properties on the subspace  $F_4(X)$  of a free topological group  $F(X)$  on a metrizable space  $X$  and obtain several characterizations of Fréchet-Urysohn and countable fun-tightness of  $F_4(X)$ .

**Definition 1.** Let  $X$  be a Tychonoff space and  $F(X)$  be the free topological group on  $X$  in the sense of Markov [1]. For each  $n \in \mathbb{N}$ ,  $F_n(X)$  stands for the subsets of  $F(X)$  formed by all words of reduced length at most  $n$ .

**Theorem 1.** Let  $X$  be a metrizable space.  $F_4(X)$  is Fréchet-Urysohn if and only if the set of non-isolated points of  $X$  is compact.

**Theorem 2.** Let  $X$  be a metrizable space.  $F_4(X)$  contains no closed copy of  $S_\omega$  if and only if either  $X$  is compact or discrete.

**Corollary 1.** Let  $X$  be a metrizable space.  $F_4(X)$  is strongly Fréchet-Urysohn (weakly first-countable, countable fan-tightness) if and only if either  $X$  is compact or discrete.

**Theorem 3.** Let  $X$  be a metrizable space.  $F_4(X)$  contains no closed copy of  $S_{\omega_1}$  if and only if  $X$  is either separable or discrete.

Keywords: Free topological groups, Fréchet-Urysohn spaces, metrizable spaces.  
2010 Mathematics Subject Classification: 54H11, 22A05, 54E35, 54D45.

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## On Mean Intuitionistic Open Sets

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In this paper, dual concepts of maximal intuitionistic open and minimal intuitionistic open sets, namely minimal intuitionistic closed and maximal intuitionistic closed sets are defined. Basic properties of these sets are investigated. Also, the notions of mean intuitionistic open and mean intuitionistic closed sets are introduced, and some features of such sets are studied. Finally, results related to disconnectedness of intuitionistic topological spaces are obtained using maximal intuitionistic open, minimal intuitionistic open and mean intuitionistic open sets. Several examples and counterexamples are given to support this study.

**Keywords:** Minimal intuitionistic closed set, maximal intuitionistic closed sets, mean intuitionistic open sets and mean intuitionistic closed sets.

**2010 Mathematics Subject Classification:** 54A05, 03F55.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Some Generalizations of Caristi Type Fixed Point Theorem on $M$ -Metric Spaces

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The Caristi fixed point theorem attracted the attention of many researcher. In this paper we investigate the Caristi fixed point theorem on  $M$ -metric spaces. First, we show that Caristi's fixed point theorem doesn't fully extend to  $M$ -metric spaces. Then, we defined two mappings so called the Caristi mapping of type (I) and the Caristi mapping of type (II) and prove a Caristi type fixed point theorem in this space.

Keywords: Caristi fixed point theorem,  $M$ -metric.

2010 Mathematics Subject Classification: 54H25, 47H10.

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## Topological Aspect of Monodromy Groupoid for a Topological Internal Groupoid

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The notion of monodromy groupoid was originally introduced by J. Pradines in [4] and has been developed by many others (e.g. [1, 2, 3]).

On the one hand, the monodromy groupoid of a topological internal groupoid in groups with operations including groups, rings, associative algebras, associative commutative algebras, Lie algebras, Leibniz algebras, alternative algebras and others is considered in [2]. On the other hand, Mucuk and Demir in [3] developed topological aspect of monodromy groupoid and proved that the monodromy groupoid of a topological group-groupoid is also a topological group-groupoid.

The aim of this paper is to extend the results of latter paper to the former case.

**Keywords:** Monodromy groupoid, internal groupoid, holonomy groupoid, universal covering.

**2010 Mathematics Subject Classification:** Primary 20L05, 57M10; Secondary 22AXX, 22A30.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## On Strong Pre-Continuity with Fuzzy Soft Sets

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We adapt strong  $\theta$ -precontinuity into fuzzy soft topology and investigate its properties. Also, the relations with the other types of continuity in fuzzy soft topological spaces are analyzed. Moreover, we give some new definitions.

Keywords: Fuzzy soft pre- $\theta$ -open, fuzzy soft strong  $\theta$ -precontinuity, fuzzy soft pre- $\theta$ -closure points, fuzzy soft pre-regular, graph of a fuzzy soft function.

2010 Mathematics Subject Classification: 54A05, 54A40, 54C10, 54B10.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Direct Sum of Barreled Locally Convex Cones

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We investigate the connections between barrel subsets and their components in direct sum cone topologies and prove that the locally convex direct sum cone of barreled cones is barreled.

Keywords: Locally convex cones, direct sums, barreledness

2010 Mathematics Subject Classification: 46A03, 20K25, 46A08.

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## Crossed Module Aspects of Monodromy Groupoids for Internal Groupoids

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The notion of monodromy groupoid was introduced by J. Pradines in [3] to generalize the standard construction of a simply connected Lie group from a Lie algebra to a construction of a Lie groupoid from a Lie algebroid and has been developed by many others.

The categorical equivalence between internal groupoids and crossed modules in groups with operations is known by [2] as a generalization of an equivalence of crossed modules within groups and group-groupoids [1].

In this work using the former equivalence and techniques of crossed modules we give a construction of the monodromy groupoid for topological internal groupoids within groups with operations including groups, rings, associative algebras, associative commutative algebras, Lie algebras, Leibniz algebras, alternative algebras and some others.

**Keywords:** Monodromy groupoid, internal groupoid, crossed module.

**2010 Mathematics Subject Classification:** Primary 20L05; 22A22; 57M10.

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## A Condition for Points and Compact Subsets of $C(X)$ to be $G_\delta$ Subsets of $\mathbb{R}^X$

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It was given in [5] a condition for points and compact subsets of  $C(X)$  to be  $G_\delta$  subsets of  $\mathbb{R}^X$ , the set of all real-valued functions defined on a topological space  $X$ . Also, it was shown in the same paper that if  $C(X)$  contains a non empty  $G_\delta$  subsets of  $\mathbb{R}^X$ , then  $X$  is the topological sum of a  $\sigma$ -compact space and a discrete space. In this work it will be shown that these two results remain valid in the framework of a set-open topology more general than the considered compact-open topology.

**Keywords:** Set-àpen topology,  $G_\delta$ sets,  $\sigma$  compact sets.

**2010 Mathematics Subject Classification:** 54C35.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September–08 September 2019, Maltepe University, Istanbul, Turkey

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## $G$ -Fuzzy Sequential Continuity in FTS

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Cakalli extended the concept of  $G$ -sequential compactness to a fuzzy topological group and introduced the notion of  $G$ -fuzzy sequential compactness, where  $G$  is a function from a suitable subset of the set of all sequences of fuzzy points in a fuzzy first countable topological space  $X$ . The aim of this paper is to investigate whether an idea like the  $G$ -fuzzy continuity can be introduced and consequently can be extended to a more general approach to fuzzy continuity in fuzzy topological spaces. In this article, we introduce the concepts of  $G$ -fuzzy sequential continuity and  $G$ -fuzzy sequential closedness in a fuzzy topological space and give some characterization theorems.

**Keywords:** Fuzzy points,  $G$ -fuzzy convergence,  $G$ -fuzzy sequential closedness,  $G$ -fuzzy sequential continuity.

**2010 Mathematics Subject Classification:** Primary: 03E72; Secondary: 40A05, 40J05.

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International Conference of Mathematical Sciences (ICMS 2019)  
 04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## $m^*$ - $g$ -Closed Sets in Minimal Spaces $(X, m)$ with Hereditary Hlasses due to Császár

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Noiri and Popa [3] have defined the minimal local function and constructed a minimal structure  $m * H$  which contains  $m$  in a hereditary minimal space  $(X, m, H)$ . Moreover the concepts of  $m$ - $H_g$ -closed sets and  $(\Lambda, m_H^*)$ -closed sets in a minimal space  $(X, m)$  with a hereditary class  $H$  are presented and investigated by Noiri and Popa [3]. In this study, we define the notions  $m^*$ - $g$ -closed sets and  $m^*$ - $H_g$ -closed sets on a minimal space  $(X, m)$  with a hereditary class  $H$  due to Császár [2] and explore some of their basic characteristics and few characterizations

Keywords:  $m^*$ - $H_g$ -closed,  $m^*$ - $g$ -closed,  $m$ - $T_{\frac{1}{2}}$ ,  $m^*$ - $T_{\frac{1}{2}}$ ,  $m^*$ - $R_1$ ,  $m$ - $R_0$ ,  $m^*$ - $R_0$ .  
 2010 Mathematics Subject Classification: 54A05, 54A10, 54D10.

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## The Shortest Length Distance and the Digital $r$ -Thickening on Digital Images

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A *digital image*  $X$  is a subset of the Cartesian product of the set of integers  $\mathbb{Z}^n$ . To study the features of  $X$  without constructing a topology on it, we impose a relation,  $\kappa$ , called *an adjacency relation* [1] on the points of it to adapt the fundamental concepts of topology such as connectedness, path connectedness, and continuity [2, 3]. Suppose  $X$  is a connected digital image,  $\kappa$  is an adjacency relation defined on it, and  $A$  is a subset of  $X$ . For a point  $x \in X$ , Boxer defined *the shortest length distance* from  $x$  to  $A$  [4]. Then the shortest length distance turns into a metric function on  $X$  by assuming  $A$  as a singleton subset of  $X$ . The main goal of this study is to measure the distance of two subsets of a connected digital image which is compatible with continuous functions. To do this, we consider this metric function on a connected digital image  $X$  and define the concept of  *$r$ -thickening* of a nonempty subset of  $X$  for a nonnegative integer  $r$  to define the distance between the subsets of  $X$ . This talk is about the recent progress of this study.

Keywords: Digital topology, Hausdorff distance, metric space.

2010 Mathematics Subject Classification: 54H99, 68R01, 68U10.

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## On Measures of Parameterized Fuzzy Compactness

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In this work, the parameterized degree of compactness of a lattice valued fuzzy soft set is described in a fuzzy soft topological space. The extended versions of the basic compactness properties known in general topology are investigated for the given notion and some other characterizations of parameterized degree of compactness are specified. Further, a generalized version of Tychonoff Theorem is proved in the product fuzzy soft topological space.

**Keywords:** fuzzy soft set, fuzzy soft topology, base and subbase, compactness degree.

**2010 Mathematics Subject Classification:** 06D72, 54A40, 54D30.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Chain Connectedness

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In this paper it is defined a chain connectedness of a set in a topological space as well as proved some results for chain connectedness. It is emphasised the equivalence between quasicomponents and components of chain connectedness.

**Keywords:** General topology, connectedness, chain connectedness, quasicomponents.

**2010 Mathematics Subject Classification:** Primary: 54D05.

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## 6.2 Analysis and Functional Analysis

Session Organizer: İbrahim Çanak

The Analysis and Functional Analysis session of the 3rd International Conference of Mathematical Sciences (ICMS 2019) aims to provide a platform for presenting and discussing the most recent developments in a wide variety of topics including ordinary and partial differential equations, integral equations, numerical analysis, operator theory, finite differences and functional equations and approximation and expansions.

There are 27 oral and 3 poster presentations submitted for this session. This session of this conference has been a perfect opportunity for participants coming from Algeria, Turkey, Iran, Morocco, Indian, Unites States and many countries to present and discuss topics in their respective research areas.

We hope that some extended abstracts presented in the conference will be published in AIP conference proceedings as well as in mathematical journals.

International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Reduced Approximation for Operator with Time in Parabolic Problems Case

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This work is devoted to singular perturbation of the parabolic equation with discontinuous coefficients for the time operator. For P1-P0 finite element, by using a reduction of the approximation order for the time differential operator, we propose a numerical method which does not have any oscillations in the neighborhood of the coefficient discontinuity. Error estimates of order two with respect to space are provided, and we have compared this method with the modified second member method (T.T. Cuc Bui, 2008). Euler explicit and implicit time schemes are proposed, and by considering a toy problem, the order one and two of convergence with respect to time and space is checked.

Keywords: Singular mass matrix, Error estimates, Degenerate operator.  
2010 Mathematics Subject Classification: 65M15, 65M60.

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## Stabilization of the Wave Equation with a Localized Memory Term and Border Friction Dissipation

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**Abstract:** This paper is devoted to the study of uniform decay of a wave equation with dynamical boundary conditions, localised memory term and frictional dampings. We prove that a localised memory term combined with frictional dissipations is strong enough, via transmission process  $u|_{\Gamma} = v$ , to assure the asymptotic stability of the whole system.

**Keywords:** Stabilization; wave equation; frictional damping; Kelvin-Voigt damping; viscoelastic material; localized damping.

**2010 Mathematics Subject Classification:** 93D15, 35L05.

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## Statistical quasi Cauchy sequences in abstract metric spaces

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In this study, we introduce a concept of statistical quasi-Cauchyness of a sequences in a cone metric space in the sense that a sequence  $(x_k)$  is statistically quasi-Cauchy if  $\lim_{n \rightarrow \infty} \frac{1}{n} |\{k \leq n : d(x_{k+1}, x_k) \geq c\}| = 0$  for each  $c \in P^0$ . It turns out that a real valued function  $f$  is uniformly continuous either on a totally bounded subset of a cone metric space  $X$  or on a connected subset of  $X$  if  $f$  preserves statistical quasi-Cauchy sequences.

**Keywords:** Statistical boundedness, statistical convergence, lacunary sequence.

**2010 Mathematics Subject Classification:** 40A05, 40C05, 46A45.

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## Existence Results of First-Order Differential Systems with Mixed Quasimonotone Nonlinearities and Integral Boundary Condition

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**Abstract:** This work is concerned with the construction of solutions for a class of first order differential systems with mixed quasimonotone nonlinearities and integral boundary conditions. We also give an example to illustrate our results.

$$\begin{cases} u'(t) = f(t, u, v), t \in J, \\ v'(t) = g(t, u, v), t \in J, \\ u(0) = \int_0^T g_1(s)u(s)ds, \\ v(0) = \int_0^T g_2(s)v(s)ds, \end{cases}$$

where  $f : J \times \mathbb{R}^2 \rightarrow \mathbb{R}$ ,  $g : J \times \mathbb{R}^2 \rightarrow \mathbb{R}$  are continuous functions,  $f$  is increasing in  $v$ ,  $g$  is decreasing in  $u$ ,  $g_i : J \rightarrow \mathbb{R}$  ( $i = 1, 2$ ) are continuous functions and change their signs and  $J = [0, T]$  with  $T > 0$ .

**Keywords:** First order differential systems; upper and lower solutions; monotone iterative technique; mixed quasimonotone nonlinearities; integral boundary conditions.

**2010 Mathematics Subject Classification:**Primary 34A34, secondary 34A12, 34B10, 47H10.

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## Steklov Eigenvalue Problem with $\alpha$ -Harmonic Solutions

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**Abstract.** Using Ljusternik-Schnirelmann principle and a new variational technic we prove that the following Steklov eigenvalue problem has infinitely many positive eigenvalue sequences.

$$\operatorname{div}(a(x, \nabla u)) = 0 \text{ in } \Omega, a(x, \nabla u) \cdot \nu = \lambda m(x) |u|^{p(x)-2} u \text{ on } \partial\Omega.$$

where  $\Omega \subset \mathbb{R}^N (N \geq 2)$  is a bounded domain of smooth boundary  $\partial\Omega$  and  $\nu$  is the outward unit normal vector on  $\partial\Omega$ . The functions  $m \in L^\infty(\partial\Omega)$ ,  $p : \bar{\Omega} \mapsto \mathbb{R}$  and  $a : \bar{\Omega} \times \mathbb{R}^N \mapsto \mathbb{R}^N$  are fulfilling appropriate conditions.

**Keywords:** Variable exponents; Steklov eigenvalue problem;  $\alpha$ -harmonic solutions; Variational methods  
**2010 Mathematics Subject Classification:** Primary 35J65, 35J60, 47J30, 58E05.

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## Second Order Sweeping Process with Almost Convex Perturbation

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In this work, we consider a differential inclusion known as the perturbed second order sweeping process for a class of subsmooth moving sets. The right-hand side contains a set-valued perturbation with almost-convex values, which is a strictly weaker condition than the convexity.

**Keywords:** Differential inclusion, sweeping process, subsmooth sets, almost convex sets.  
**2010 Mathematics Subject Classification:** 34A60; 49J53.

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## Singular Degenerate Normal Differential Operators for First-Order

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In this work, all normal extensions of the minimal operator generated by degenerate linear differential expressions for first order with selfadjoint operator coefficients in the Hilbert space of Hilbert space valued functions defined in separated left-right semi-axes in terms of boundary values are investigated. Also, the spectrum structure of any normal extension is examined.

**Keywords:** Degenerate differential operator, Formally normal and normal operator, Minimal and maximal operators, Extension, Spectrum of an operator.

**2010 Mathematics Subject Classification:** 47A20, 47A10.

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## Existence of Solutions for Delay Dynamic Equations on Time Scales

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In this work, we study the existence of positive solutions for a dynamic equations on time scales. The main tool employed here is the Schauder's fixed point theorem. The asymptotic properties of solutions are also treated. Three examples are also given to illustrate this work.

Keywords: Schauder fixed point theorem, Dynamic equations, Time scales.

2010 Mathematics Subject Classification: 34K13, 34A34.

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## Optimization of Second Order Evolution Differential Inclusions Problem with Phase Constraints

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In this paper, for a problem second order evolution differential inclusions with phase constraints the optimality conditions are derived. Necessary and sufficient conditions ensuring the existence of a solution to the problems for evolution differential inclusions of second order have been studied in recent years. There are limited number of articles devoted to the optimization problem of second order evolution differential inclusions with phase constraints. We apply optimality conditions to problems with geometric constraints and conditions for second order discrete inclusions are proved. We use Locally Dual Mapping definition to derive necessary and sufficient conditions for the optimality of the discrete approximation problem. Passing to the limit, sufficient conditions to the optimal problem are established.

Keywords: Second order differential inclusion, Locally dual mapping, evolution.  
2010 Mathematics Subject Classification: 49K20, 49K15, 90C31.

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## Global Behavior of some Difference Equations

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Here we investigate the boundedness, the global stability, the rate of convergence and the periodicity for the solutions of the difference equation

$$x_{n+1} = \delta_n + \frac{ax_n}{x_{n-1}} + \frac{bx_{n-1}}{x_n}, \quad n = 0, 1, \dots, \quad (I)$$

where  $a, b, x_{-1}$  and  $x_0$  are arbitrary positive real numbers and  $\{\delta_n\}$  is a sequence of real numbers. Eq.(I) has the unique positive equilibrium point  $\bar{x} = \delta + a + b$ .

The following are some of the obtained results presented in this study:

**Theorem:** Assume that  $2|a - b| < a + b + \delta$ . Then the positive equilibrium point of Eq.(I) is locally asymptotically stable.

**Theorem:** Every solution of Eq.(I) is bounded and persists if  $\delta > a + b$ .

**Theorem:** The equilibrium point  $x$  is global attractor of Eq.(I) if one of the following statements holds:

1.  $b < a < 3b + \delta$  and  $\bar{x}(\delta^2 + a^2) \geq a\delta(\delta + b) + \bar{x}(2a\delta + b^2)$ .
2.  $3a + \delta > b \geq a$  and  $\bar{x}(\delta^2 + b^2) \geq b\delta(\delta + a) + \bar{x}(2b\delta + a^2)$ .

**Theorem:** Consider  $2|a - b| < a + b + \delta$ , then the equilibrium point  $x$  is a global attractor of Eq.(I).

**Theorem:** If  $b > 3a + \delta$ , then the period-two solution of Eq.(I) is locally asymptotic stability.

Similar studies were given in (Elsayed 2015), (Kadry 2016) and (Özkan 2014).

Keywords: Global stability, Periodic solutions, Difference equations

.2010 Mathematics Subject Classification: 39A10.

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## Norm and Almost Everywhere Convergence of Convolution Powers

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Let  $G$  be a locally compact abelian group with the dual group  $\Gamma$ ,  $M(G)$ , the measure algebra of  $G$ , and  $M_r(G)$ , the largest regular subalgebra of  $M(G)$ . For a power bounded measure  $\mu \in M(G)$ , we put  $\mathcal{F}_\mu = \{\gamma \in \Gamma : \widehat{\mu}(\gamma) = 1\}$  and  $\mathcal{E}_\mu = \{\gamma \in \Gamma : |\widehat{\mu}(\gamma)| = 1\}$ , where  $\widehat{\mu}$  is the Fourier-Stieltjes transform of  $\mu$ .

Let  $(\Omega, \Sigma, m)$  be a  $\sigma$ -finite positive measure space and let  $\Theta = \{\theta_g\}_{g \in G}$  be an action of  $G$  in  $(\Omega, \Sigma, m)$  by invertible measure preserving transformations. Any action  $\Theta$  induces a representation  $\mathbf{T} = \{T_g\}_{g \in G}$  of  $G$  on  $L^p(\Omega)$  ( $1 \leq p < \infty$ ) by invertible isometries, where  $(T_g f)(\omega) = f(\theta_g \omega)$ . If  $\Theta$  is continuous, then for any  $\mu \in M(G)$ , we can define a bounded linear operator on  $L^p(\Omega)$  ( $1 \leq p < \infty$ ) associated with  $\mu$ , denoted by  $\mathbf{T}_\mu$ , which integrates  $T_g$  with respect to  $\mu$ .

**Theorem.** Let  $\mu \in M_r(G)$  be power bounded and  $1 < p < \infty$ . If  $\mathcal{F}_\mu = \mathcal{E}_\mu$ , then the sequence  $\{\mathbf{T}_\mu^n f\}$  converges strongly for every  $f \in L^p(G)$ .

Keywords: Abelian group, measure algebra,  $L^p$ -space, convergence.

2010 Mathematics Subject Classification: 47D03; 46J05; 43A65.

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## An Extreme Point Theorem

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In this talk, we characterize the extreme points of the collection consisting of all positive linear operators defined between two Archimedean semiprime  $f$ -algebras having weak order units.

Keywords: lattice homomorphism,  $f$ -algebra.

2010 Mathematics Subject Classification: 47B38.

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## Nonlocal Elliptics Problems with Hardy Potential Term

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Let  $\Omega \subset \mathbb{R}^N$  be a bounded regular domain of  $\mathbb{R}^N$  and  $N > 2s$ . We consider

$$(P) \begin{cases} (-\Delta)^s u = \frac{u^q}{d^{2s}} & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{in } \mathbb{R}^N \setminus \Omega, \end{cases}$$

where  $0 < q \leq 2^* - 1$ ,  $0 < s < 1$  and  $d(x) = \text{dist}(x, \partial\Omega)$ . The main goal of this paper is to analyze the existence of solution to problem (P) according to the value of  $s$  and  $q$ .

The main result is the following,

**Theorem 1.** Assume that  $\Omega$  is bounded regular domain of  $\mathbb{R}^N$  and  $0 < s < 1$ , then, for all  $q > 0$ , problem (P) has bounded distributional positive solution in the sense of Definition 1 such that  $u(x) \geq Cd(x)$  in  $\Omega$

**Definition 1.** We say that  $u \in L^1(\Omega)$  is a distributional solution to problem (P) if for all  $\psi \in C_0^\infty(\Omega)$ , we have  $\int_\Omega u(-\Delta)^s \psi dx = \int_\Omega h \psi dx$ , where  $h = \frac{u^q}{d^{2s}}$ .

Keywords: Hardy inequality, Nonlinear elliptic problems, singular weight.

2000 Mathematics Subject Classification: 35D05, 35D10, 35J20, 35J25, 35J70.

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## Some estimates in homogeneous function spaces

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We are interested to study the convergence of the series of type  $\sum_{j \in \Lambda} u_j$  (where  $\Lambda \subseteq \mathbb{N}$  or  $\mathbb{Z}$ ), on some function spaces, as Besov spaces  $B^{s,q}(E)$ , Triebel-Lizorkin spaces  $F^{s,q}(E)$ , and their versions homogeneous defined the space  $\mathcal{S}'_{\infty}(\mathbb{R}^n)$  of tempered distributions modulo polynomials, where  $E$  is a Banach distribution space (B.D.S) on  $\mathbb{R}^n$ .

So we will prove essentially the following result:

**Theorem 1.** Let  $E$  be a B.D.S in  $\mathcal{S}'_{\infty}(\mathbb{R}^n)$  translation invariance. Let  $a, b$  be such that  $0 < a < b$ . Let  $(u_j)_{j \in \mathbb{Z}}$  be a sequence in  $\mathcal{S}'(\mathbb{R}^n)$  such that

- $\hat{u}_j$  is supported by the annulus  $a2^j \leq |\xi| \leq b2^j$ ,
- $|u_j(x)| \leq c \max(2^{jN}, 2^{-jN})(1 + |x|)^N$  for some  $N \in \mathbb{N}$ ,
- $A := (\sum_{j \in \mathbb{Z}} (2^{js} \|u_j\|_E)^q)^{1/q} < \infty$  (resp.  $A := \|(\sum_{j \in \mathbb{Z}} (2^{js} |u_j|)^q)^{1/q}\|_E < \infty$ ).

1- Then the series  $\sum_{j \in \mathbb{Z}} u_j$  converges in  $\mathcal{S}'_{\infty}(\mathbb{R}^n)$  to an element  $u$  which satisfies  $\|u\|_{\dot{B}^{s,q}(E)} \leq cA$  (resp.  $\|u\|_{\dot{F}^{s,q}(E)} \leq cA$ ).

2- The same conclusion holds: for  $a = 0$  if  $s > 0$ , for  $b = +\infty$  if  $s < 0$ .

**Keywords:** Besov spaces, Triebel-Lizorkin spaces, Homogeneous spaces

**2010 Mathematics Subject Classification:** 46E35, 47H30

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## Weighted Stepanov Pseudo Almost Periodic Functions on Time Scales

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In this work, by using the measure theory on time scales, we propose a new class of functions called weighted Stepanov-like pseudo almost periodic functions on time scales, which generalize in a natural fashion the class of almost periodic functions and its various extensions. Basic properties of these new functions are studied including a composition result. To illustrate our abstract results, we study the existence and uniqueness of pseudo almost periodic solutions to some classes of nonautonomous differential equations involving weighted  $S^p$ -pseudo almost periodic forcing terms on time scales.

**Keywords:** Weighted pseudo almost periodicity; time scales.

**2010 Mathematics Subject Classification:** 34G20, 35B10, 37B55, 47D06.

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## On a Study of New Class of $q$ -Fractional Operator

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New class of  $q$ -fractional integral operator is introduced by using iterated Cauchy integral on Jackson integral. It is shown that our new class of operator unify all  $q$ -fractional integral operators and  $q$ -analogue of Hadamard integral operator is investigated. Semi group properties of integral operator is studied and corresponding  $q$ -fractional derivative operator is introduced.

**Definition 1**  $q$ -fractional integral operator  $J_{p,q}^\alpha (f(x))$  defined as

$$J_{p,q}^\alpha (f(x)) = \frac{([p]_q)^{1-\alpha}}{\Gamma_{q^p}(\alpha)} \int_0^x w^{p-1} f(w) (x^p - (wq)^p)_{q^p}^{(\alpha-1)} d_q w.$$

Keywords:  $q$ -fractional difference operator,  $(p, q)$ -absolutely continuous, fractional integral operator.  
 2010 Mathematics Subject Classification: 34A12.

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## An Inequality for Self Reciprocal Polynomials

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Let  $\mathcal{P}_n$  be the class of all polynomials of degree at most  $n$ . Polynomials  $f \in \mathcal{P}_n$  which satisfy the condition  $z^n f(1/z) \equiv f(z)$  are called self-reciprocal and form the sub-class  $\mathcal{P}_n^*$  of  $\mathcal{P}_n$ . For any  $\rho > 0$ , let  $\mathcal{M}_\infty(f; \rho) := \max_{|z|=\rho} |f(z)|$  and  $\mathcal{M}_p(f; \rho) := \left( \frac{1}{2\pi} \int_{-\pi}^{\pi} |f(\rho e^{i\theta})|^p d\theta \right)^{1/p}$ ,  $0 < p < \infty$ . If  $f \in \mathcal{P}_n$  then  $\mathcal{M}_p(f'; \rho) \leq n\rho^{n-1} \mathcal{M}_p(f; 1)$  for any  $p > 0$  and  $\rho \geq 1$ , whereas, if  $f \in \mathcal{P}_n^*$  then  $\mathcal{M}_p(f'; \rho) \leq (n/2)\rho^{n-1} \mathcal{M}_p(f; 1)$  for any  $p > 0$  and  $\rho \geq 1$ . Lately, it has been noted that at least for  $p \geq 1$ , there exists a positive number  $\rho_n$  strictly less than 1 such that  $\mathcal{M}_p(f'; \rho) \leq n\rho^{n-1} \mathcal{M}_p(f; 1)$  for  $\rho \geq \rho_n$  if  $f \in \mathcal{P}_n$ . By analogy, it has been asked if there was a positive number  $\rho_n^* < 1$  such that  $\mathcal{M}_p(f'; \rho) \leq (n/2)\rho^{n-1} \mathcal{M}_p(f; 1)$  for all  $\rho \geq \rho_n^*$  and any  $f \in \mathcal{P}_n^*$ . We propose to discuss this question.

Keywords: Polynomials, Bernstein's inequality, Zygmund's inequality.

2010 Mathematics Subject Classification: 30D15, 30D99, 41A17

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## On Property $(UW_E)$ under Functional Calculus

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**Abstract.** In this note we study the properties  $(UW_E)$  and  $(W_E)$  are the variant of Weyl's theorem. We establish for a bounded linear operator defined on a Banach space several sufficient and necessary conditions for which property  $(UW_E)$  holds. We also relate this property with property  $(UW_\Pi)$ , and we show that if  $T$  is polaroid and  $T$  satisfies property  $(UW_E)$  then properties  $(UW_E)$  and  $(UW_\Pi)$  are equivalent under functional calculus  $f(T)$ .

**Keywords:** Property  $(UW_\Pi)$ , property  $(UW_E)$ , B-Weyl spectrum.  
**2010 Mathematics Subject Classification:** Primary 47A53, 47A10, 47A11.

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## Pseudo Almost Periodic Generalized Functions

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The aim of this work is to introduce and to study an algebra of pseudo almost periodic generalized functions containing the classical pseudo almost periodic functions as well as pseudo almost periodic distributions.

Keywords: Pseudo almost periodic functions, Colombeau algebra, Pseudo almost periodic generalized functions.  
2010 Mathematics Subject Classification: 46F30, 42A75.

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## Global Existence and Uniqueness of Solutions to a Parabolic Haptotaxis Model

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We are dealing with the the models suggested by M.A.J and all Chaplain and all, [2, 3]. We prove existence and uniqueness of a local in time solution to the problem. Next, the problem is whether the solution blows-up in finite time or not. The answer depends on the choice of the initial data. Indeed, we can find in the literature examples of solutions blowing-up in finite time under appropriate initial data. In this work, we give sufficient conditions avoiding the blow-up of the solution so that the solution is global in time.

**Keywords:** Quasilinear parabolic equations, Nonlinear parabolic problems, Reaction-diffusion systems, Chemotaxis-Haptotaxis.

**2010 Mathematics Subject Classification:** 35K59, 35K57, 35K60.

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## On the $\Delta_g$ -statistical convergence of the function defined time scale

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We have introduced the concept  $\Delta_g$ -convergence of a subset of the time scale  $\mathbb{T}$  with the help of modulus function  $g$ , and give the definitions of  $\Delta_g$ -convergence and  $\Delta_g$ -Cauchy for a real valued function  $f$  defined on the time scale  $\mathbb{T}$ . Later, we discussed the connection between classical convergence,  $\Delta$ -convergence and  $\Delta_g$ -convergence. In addition to, we have seen that  $\Delta$ -convergent function  $f$  is not  $\Delta_g$ -convergent.

**Keywords:** Time scale, modulus function, statistical convergence.

**2010 Mathematics Subject Classification:** 34N05, 40A35.

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## A Minimization Algorithm for Limit Extremal Problems on Convex Compactum

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In an extremal problem, instead of  $f(x)$  one has a sequence  $f_n(x)$  of functions approximating in some sense  $f(x)$  and on the basis of which one has to find an extremum of  $f(x)$ , such problems are usually called limit extremal problems. In this study, a minimization algorithm for limit extremal problems is proposed under some certain constraints. Let the set  $X \subset \mathbb{R}^n$  be a nonempty convex compactum, i.e.,  $X$  is closed, bounded and convex. We will consider the following extremal problem

$$\min_{x \in X} f(x)$$

where  $f : X \rightarrow \mathbb{R}$  is a function defined as a limit of a sequence of functions  $f(x) = \lim_{n \rightarrow \infty} f_n(x)$  on  $X$ . Our aim is to construct a minimization algorithm for  $f(x)$ .

Keywords: Limit extremal problems, convex optimization.

2010 Mathematics Subject Classification: 65K05, 90C25.

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## First Order Maximally Dissipative Singular Differential Operators

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In this work, using the Calkin-Gorbachuk method, the general form of all maximally dissipative extensions of the minimal operator generated by first order linear multipoint symmetric singular differential-operator expression in the direct sum of Hilbert space of vector-functions has been found. Later on, the structure of spectrum of these extensions is researched. Finally, the results are supported by application.

**Keywords:** Dissipative differential operator, selfadjoint differential operator, deficiency index, space of boundary values, spectrum.

**2010 Mathematics Subject Classification:** 47A10, 47B25.

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## Schatten-von Neumann Characteristic of Tensor Product Operators

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In this work, the relations between Schatten-von Neumann property of the tensor product of operators and Schatten-von Neumann property of its coordinate operators are studied.

**Keywords:** Tensor product of Hilbert spaces and operators, compact operator, Schatten-von Neumann operator classes, singular number.

**2010 Mathematics Subject Classification:** 47A80, 47B06, 47B10.

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## Some Properties of Solutions to Dynamical Systems

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We consider the following differential inclusion with discontinuous right-hand sides:

$$\dot{x}(t) \in \Gamma(t, x(t)) \quad (1)$$

where  $\Gamma$  is a set-valued function (i.e., multifunction) which associates with any point  $x \in \mathbb{R}^n$ , a set  $\Gamma(t, x) \subset \mathbb{R}^n$  and  $x(\cdot)$  is an absolutely continuous (*AC*) function from  $[0, T]$  to  $\mathbb{R}^n$ . We say that the function  $x(\cdot) \in AC([0, T], \mathbb{R}^n)$  with  $x(0) = x_0$  satisfying (1) almost everywhere is a solution to the system (1) with the initial condition  $x_0$ .

In this paper, we derive some properties concerning set-valued functions satisfying one-sided Lipschitz condition and solutions to the system (1) in a weighted space.

**Keywords:** set-valued functions, one-sided Lipschitz condition, differential inclusion.

**2010 Mathematics Subject Classification:** 28B20, 34A60.

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## Sufficient Conditions of Optimality for Free Time Optimization of Third Order Differential Inclusions

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This talk deals with the free time optimization problem described by third order differential inclusions ( $P_C$ ) with endpoint constraints. In order to construct the optimality conditions for our problem ( $P_C$ ), we obtain the optimality conditions for the discrete-approximation problem associated with the auxiliary differential problem ( $P_A$ ) given by third order convex differential inclusions. Formulation of optimality conditions for problem ( $P_A$ ) plays a substantial role in incorporating the Euler-Lagrange and Hamiltonian type inclusions and moreover by using distinctive  $t_1$ -attainability conditions on the initial sets, the sufficient conditions for our main problem ( $P_C$ ) are established.

Keywords: Differential inclusion, free time, attainability.

2010 Mathematics Subject Classification: 49K21,34A60.

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## Shechter Spectra and Relatively Demicompact Linear Relations

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### Abstract

In this paper, we denote by  $\mathcal{L}$  the block matrix linear relation, acting on the Banach space  $X \oplus Y$ , of the form

$$\mathcal{L} = \begin{pmatrix} A & B \\ C & D \end{pmatrix},$$

where  $A, B, C$  and  $D$  are four linear relations with dense domains. We first try to determine the conditions under which a block matrix linear relation becomes a demicompact block matrix linear relation.

Second, we study Shechter spectra using demicompact linear relations and relatively demicompact linear relations.

**Keywords:** relatively demicompact linear relation, demicompact block matrix linear relation.

**2010 Mathematics Subject Classification:** 47A06.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Rate of Convergence by Phillips Operators Involving Appell Polynomials

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In this talk, we investigate approximation properties of summation-integral type Phillips operators involving Appell polynomials with the help of modulus of continuity. Moreover, we show the rate of convergence for these operators to certain functions by illustrative graphics.

Keywords: Phillips operators, Appell polynomials, degree of approximation.

2010 Mathematics Subject Classification: 41A36, 41A35, 41A30.

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## On the rigidity part of Schwarz Lemma at the boundary

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### Abstract

We consider the rigidity part of Schwarz Lemma. Let  $f$  be a holomorphic function in the unit disc  $D$  and  $|\Re f(z)| < 1$  for  $|z| < 1$ . We generalize rigidity of holomorphic function and provide sufficient conditions on the local behaviour of  $f$  near a finite set of boundary points that needs  $f$  to be a finite Blaschke product. For a different version of the rigidity theorems of D. Burns-S.Krantz and D. Chelst, we present some more general results used the bilogarithmic concave majorants. The strategy of these results relies on a special version of Phragmen-Lindelöf principle and Harnack inequality.

**Keywords:** Holomorphic function, Bilogarithmic concave majorant, Harnack inequality, Phragmen-Lindelf principle

**2010 Mathematics Subject Classification:** Primary 30C80; Secondary 32A10

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## The Transmission Problem for the Laplace Operator on a Domain with a Cuspidal Point

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We shall study the transmission problem for the Laplace operator subject to Dirichlet boundary conditions in a plane domain  $\Omega$  with an external cusp,  $\Omega$  being divided into two subdomains  $\Omega_1$  and  $\Omega_2$  separated by a straight interface. We look for a solution in the framework of  $L^p$ -Sobolev spaces,  $1 < p < +\infty$ . We prove that  $u \in H_0^1(\Omega)$  solution of  $-\operatorname{div}(p\nabla u) = f$  with  $f$  in  $L^p(\Omega)$  and  $p = p_i$  on  $\Omega_i$ ,  $i = 1, 2$ , ( $p_1, p_2$  are two positive real numbers, supposed to be different), is piecewise in  $W^{2,p}$ .

Keywords: transmission problem, regularity, cuspidal domains.  
2010 Mathematics Subject Classification: 35J25, 35R05, 35A20

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## 6.3 Sequences, Series, Summability

Session Organizer: Hacer Şengül

The Symposium "Sequences, Series, Summability" is organized in ICMS 2019, Maltepe University, Istanbul, Turkey, 04th - 08th September, 2019. Sequence spaces and series are widely applied to various other branches of functional analysis, e.g., the theory of functions, summability theory, the theory of locally convex spaces, nuclear spaces, and matrix transformations. The theory of summability is used in many areas of analysis and applied mathematics. The engineer or physicist who works with Fourier series, Fourier transforms or analytic continuation will find the concepts of summability theory extremely useful for their research.

This symposium is organized to bring together scientists who have worked in the fields of sequences, series and summability. In this section, results will be given about how the sequence spaces and series are used in different spaces.

However, activities will include the discussion of different methods of summabilities in computer sciences and engineering, where will be presented theoretical studies and experimental results. The session interest subjects include, but are not limited to:

- 1) Sequence Spaces
- 2) Tauberian Theorems
- 3) General Matrix Methods
- 4) Special Summability Methods
- 5) Approximation Theory
- 6) Logarithmic Summability
- 7) Absolute Riesz Summability

International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## On a Summation by the Abel Method of a System of Root Functions for a Nonlocal Problem with Integral Conditions

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**Abstract.** In this paper, we consider an ordinary differential operator with integral conditions without measure atomicity at the endpoints of the interval containing derivatives of the unknown functions. We obtain the result on Abel summability of a system of root functions. Note that a similar problem was treated in the case integral conditions containing of the unknown functions in [1].

**Keywords:** Ordinary differential equation, integral condition, Abel summability.  
**2010 Mathematics Subject Classification:** 34L10 , 34B10, 40A30.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## On Rough Convergence of Triple Sequences

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In this paper we define and study rough convergence of triple sequences, the set of rough limit points of a triple sequence. Also investigate the relations between the set of cluster points and the set of rough limit points of a triple sequence.

**Keywords:** Triple sequences, rough convergence, rough limit points.

**2010 Mathematics Subject Classification:** 40A05, 40C99, 40G05.

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International Conference of Mathematical Sciences (ICMS 2019)  
 04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Triple sequence spaces of Intuitionistic Rough $I$ -convergence defined by compact Bernstein operator

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This paper is to introduce the triple sequence spaces of intuitionistic rough  $I$ -convergent of  $B_{\Lambda^3(\mu, \gamma)}(f, x, T)$  and  $B_{\chi^3(\mu, \gamma)}(f, x, T)$  are defined by compact Bernstein operator and study the topology general properties.

**Keywords:** Triple sequences, rough convergence, closed and convex, cluster points and rough limit points, compact, Bernstein polynomials, Intuitionistic  $I$ -convergence.

**2010 Mathematics Subject Classification:** 40F05, 40J05, 40G05.

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## Necessary and Sufficient Tauberian Conditions Under Which Convergence Follows from $A^{r,\delta}$ Summability

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Let  $x = (x_{mn})$  be a double sequence of real or complex numbers. The  $A^{r,\delta}$ -transform of a sequence  $(x_{mn})$  is defined by

$$(A^{r,\delta}x)_{mn} = \sigma_{mn}^{r,\delta}(x) = \frac{1}{(m+1)(n+1)} \sum_{j=0}^m \sum_{k=0}^n (1+r^j)(1+\delta^k)x_{jk}, \quad 0 < r, \delta < 1$$

We say that  $(x_{mn})$  is  $(A^{r,\delta}, 1, 1)$  summable to  $l$  if the sequence  $(\sigma_{mn}^{r,\delta}(x))$  has a finite limit  $l$ . It is known that if  $\lim_{m,n \rightarrow \infty} x_{mn} = l$  and  $(x_{mn})$  is bounded, then the limit  $\lim_{m,n \rightarrow \infty} \sigma_{mn}^{r,\delta}(x) = l$  exists. But the inverse of this implication is not true in general. Our aim is to obtain necessary and sufficient conditions for  $(A^{r,\delta}, 1, 1)$  summability method under which the inverse of this implication holds. Following Tauberian theorems for  $(A^{r,\delta}, 1, 1)$  summability method, we also define  $A^r$  and  $A^\delta$  transformations of double sequences and obtain Tauberian theorems for the  $(A^{r,\delta}, 1, 0)$  and  $(A^{r,\delta}, 0, 1)$  summability methods.

**Keywords:**  $(A^{r,\delta}, 1, 1)$ ,  $(A^{r,\delta}, 1, 0)$  and  $(A^{r,\delta}, 0, 1)$  summability methods, Pringsheim's convergence, slow decrease and slow oscillation in different senses, Tauberian conditions and theorems.

**2010 Mathematics Subject Classification:** 40E05, 40G05.

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## Characterization of the Compact Operators on the Class $(bv, bv_k^\theta)$

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The space  $bv$ , the set of all bounded variation sequences, has an important role in the summability theory. In recent study, this space has been extended to the space  $bv_k^\theta$  and some matrix class on this space has been characterized [2]. In the present paper, for  $1 \leq k < \infty$ , computing Hausdorff measure of non-compactness, we characterize compact operators in the class  $(bv, bv_k^\theta)$ , where  $\theta$  is a sequence of positive numbers.

Keywords: Sequence spaces; matrix transformations;  $bv_k^\theta$  spaces.

2010 Mathematics Subject Classification: 40C05, 40D25, 40F05, 46A45.

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## Certain Matrix Characterizations Related to the Difference Spaces

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The spaces  $l_\infty(\Delta_q)$ ,  $c(\Delta_q)$  and  $c_0(\Delta_q)$  are introduced and studied by Sarıgöl in [4] where  $q < 1$ . The purpose of the present study is to give some matrix characterizations related these spaces such that  $(E, F'_q)$  and  $(E'_q, F'_q)$  where  $E \in \{l_\infty, c, c_0\}$  and  $E'_q$  is one of the above difference sequence spaces. Also, characterizations of the matrix classes  $(E'_q, bs)$  and  $(E'_q, cs)$  have been given as a result.

**Keywords:** Difference matrix, matrix transformation, sequence space.

**2010 Mathematics Subject Classification:** 40C05, 40F05, 46A45, 46B45.

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## On Some Properties of a New Paranormed Space $|C_{-1}|(p)$

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Recently, Hazar and Sarigöl have defined and studied the series space  $|C_{-1}|_k$  for  $1 \leq k < \infty$  in [2]. The aim of this study is to introduce a new paranormed space  $|C_{-1}|(p)$ , where  $p = (p_k)$  is a bounded sequence of positive real numbers, which extends the results of Hazar and Sarigöl [2]. Further, we investigate topological properties of this paranormed space.

**Keywords:** Paranormed sequence spaces, absolute summability, Cesàro means.

**2010 Mathematics Subject Classification:** 40C05, 40F05, 46A45.

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International Conference of Mathematical Sciences (ICMS 2019)  
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## Matrix operators Involving the Space $bv_k^\theta$

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In the present paper, for  $1 < k < \infty$ , determining the  $\beta$  dual of the space of  $bv_k^\theta$  we characterize the matrix classes  $(bv_k^\theta, bv)$ , where  $\theta$  is a sequence of positive numbers and  $bv_k^\theta = \left\{ x \in w : \left( \theta_v^{1/k^*} \Delta x_v \right) \in \ell_k \right\}$ .

Keywords: Sequence spaces, matrix transformations,  $bv_k^\theta$  spaces.

2010 Mathematics Subject Classification: 40C05, 40D25, 40F05, 46A45.

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04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Some Tauberian Theorems for $(C, 1, 1)$ Summability Method by Regularly Generated Integrals

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We introduce the concept of regularly generated integrals for improper double integrals. We prove several Tauberian theorems for the  $(C, 1, 1)$  summability method of improper double integrals by using regularly generated integrals.

**Keywords:** Tauberian theorems,  $(C, 1, 1)$  summability method, improper double integral, regularly generated integrals.

**2010 Mathematics Subject Classification:** 40A10, 40C10, 40E05.

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## Logarithmic Summability of Integrals on $[1, \infty)$

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Moricz [2] characterized  $(C, 1)$  summability of integrals by convergence of another integral. In this work, we extend this result to logarithmic summability  $(L, 1)$  of integrals.

Keywords: Tauberian theorems, logarithmic summability  $(L, 1)$ , improper integrals.

2010 Mathematics Subject Classification: 40A10, 40E05.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Lacunary $A$ -Statistical Convergence and Lacunary Strong $A$ -Convergence of Order $(\alpha, \beta)$ with Respect to a Modulus

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In this paper, the definitions of lacunary strong  $A$ -convergence of order  $(\alpha, \beta)$  with respect to a modulus and lacunary  $A$ -statistical convergence of order  $(\alpha, \beta)$  are given. We study some connections between lacunary strong  $A$ -convergence of order  $(\alpha, \beta)$  with respect to a modulus and lacunary  $A$ -statistical convergence of order  $(\alpha, \beta)$ . Interesting results are obtained.

Keywords: Modulus function, statistical convergence, lacunary sequence.  
2010 Mathematics Subject Classification: 40A05, 40C05, 46A45.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Deferred Statistical Convergence of Order $\alpha$ in Topological Groups

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In this paper, the concept of deferred statistical convergence of order  $\alpha$  is generalized to topological groups, and some inclusion relations between the set of all statistically convergent sequences of order  $\alpha$  in topological groups and the set of all deferred statistically convergent sequences of order  $\alpha$  in topological groups are given.

Keywords: Topological groups, statistical convergence, deferred statistical convergence.

2010 Mathematics Subject Classification: 40A05, 40C05, 46A45.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Lacunary $d$ -Statistical Convergence and Lacunary $d$ -Statistical Boundedness in Metric Spaces

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In this study, using a lacunary sequence we introduce the concepts of lacunary  $d$ -statistically convergent sequences and lacunary  $d$ -statistically bounded sequences in general metric spaces.

**Keywords:** Statistical boundedness, statistical convergence, lacunary sequence.

**2010 Mathematics Subject Classification:** 40A05, 40C05, 46A45.

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## Rho Statistical Convergence of Order Beta

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A sequence  $(\alpha_k)$  of points in  $\mathbb{R}$ , the set of real numbers, is called  $\rho$ -statistically convergent to an element  $\ell$  of  $\mathbb{R}$  of order  $\beta$  if

$$\lim_{n \rightarrow \infty} \frac{1}{\rho_n^\beta} |\{k \leq n : |\alpha_k - \ell| \geq \varepsilon\}| = 0$$

for each  $\varepsilon > 0$ , where  $\rho = (\rho_n)$  is a non-decreasing sequence of positive real numbers tending to  $\infty$  such that  $\limsup_n \frac{\rho_n}{n} < \infty$ ,  $\Delta\rho_n = O(1)$ , and  $\Delta\alpha_n = \alpha_{n+1} - \alpha_n$  for each positive integer  $n$ . A real-valued function defined on a subset of  $\mathbb{R}$  is called  $\rho$ -statistically ward continuous if it preserves  $\rho$ -statistical quasi Cauchy sequences where a sequence  $(\alpha_k)$  is defined to be  $\rho$ -statistically quasi-Cauchy if the sequence  $(\Delta\alpha_k)$  is  $\rho$ -statistically convergent to 0. We obtain results related to  $\rho$ -statistical ward continuity,  $\rho$ -statistical ward compactness, ward continuity, continuity, and uniform continuity.

**Keywords:** Sequences, series, summability, continuity.

**2010 Mathematics Subject Classification:** 40A05, 40A35, 26A15.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Abel Statistical Convergence in Metric Spaces

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In this study, we investigate the concepts of Abel statistical convergence and Abel statistical quasi Cauchy sequences. A function  $f$  from a subset  $E$  of a metric space  $X$  into  $X$  is called Abel statistically ward continuous if it preserves Abel statistical quasi Cauchy sequences, where a sequence  $(x_k)$  of point in  $E$  is called Abel statistically quasi Cauchy if  $\lim_{x \rightarrow 1^-} (1-x) \sum_{k: d(x_{k+1}, x_k) \geq \varepsilon} x^k = 0$  for every  $\varepsilon > 0$ . Some other types of continuities are also studied and interesting results are obtained.

**Keywords:** Abel statistical convergence, compactness, continuity.  
**2010 Mathematics Subject Classification:** 40A05, 40C05, 46A45.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Tauberian Theorems for the Weighted Mean Summability of Integrals on $[1, \infty)$

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In this work, we obtain some Tauberian theorems for the weighted mean method of integrals on  $[1, \infty)$ . As special cases, we give some Tauberian theorems for Cesàro summability of integrals on  $[1, \infty)$ .

**Keywords:** Tauberian conditions and theorems, weighted mean method of integrals, regular varying functions of positive index, slowly decreasing functions, slowly oscillating functions.

**2010 Mathematics Subject Classification:** 40A10, 40E05, 40G05.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Category Theoretical View of I-Cluster and I-Limit Points for Ideals $I$ with the Baire Property

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We study the concept of I-cluster and I-limit points of a sequence, where  $I$  is an ideal with the Baire property. We obtain the relationship between I-cluster and I-limit points of subsequences of a given sequence in the sense of category. Our main result is

**Theorem** Suppose  $s$  is a bounded sequence,  $L$  the set of its limit points and  $I$  is an ideal with the Baire property. Then the set of its subsequences with the same set of I-cluster points as  $s$  is co-meager if and only if all elements of  $L$  are I-cluster points of  $s$ , and is meager otherwise. The analogous statement also holds if I-limit points are in place of I-cluster points.

Keywords: Ideal convergence, subsequences, I-cluster and I-limit points.

2010 Mathematics Subject Classification: 40G99, 28A12.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Deferred Statistical Convergence and Strongly Deferred Summable Functions

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The main purpose of this paper is to introduce and investigate the concepts of deferred strong summability and deferred statistical convergence of real-valued functions which are measurable (in the Lebesgue sense) in the interval  $(1, \infty)$ . Some relations between deferred strong summability and deferred statistical convergence of real-valued functions which are measurable (in the Lebesgue sense) in the interval  $(1, \infty)$  are also given.

Keywords: Statistical convergence, measurable function, deferred Cesàro mean, deferred statistical convergence.  
2010 Mathematics Subject Classification: 40A05, 40C05, 46A45.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Lacunary Statistical Convergence of Difference Sequences of Fractional Order

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The idea of statistical convergence was given by Zygmund [1] in the first edition of his monograph published in Warsaw in 1935. Over the years and under different names statistical convergence was discussed in the theory of Fourier analysis, Ergodic theory, Number theory, Measure theory, Trigonometric series, Turnpike theory and Banach spaces. In this study we introduce the concepts of  $\Delta^\alpha$ -lacunary statistical convergence with the fractional order of  $\alpha$ ,  $\alpha \in \mathbb{R}$  and  $\Delta^\alpha$ -lacunary strongly convergence with the fractional order of  $\alpha$ ,  $\alpha \in \mathbb{R}$ , and examine some properties of these sequence spaces. We also establish some connections between  $\Delta^\alpha$ -lacunary strongly convergence of fractional order of  $\alpha$  and  $\Delta^\alpha$ -lacunary statistical convergence of fractional order of  $\alpha$ .

**Keywords:** Difference sequence, statistical convergence, lacunary sequence.

**2010 Mathematics Subject Classification:** 40A05, 40C05, 46A45.

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## Application on Local Properties of Factored Fourier Series

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In this paper, we have generalized the main theorem concerning local property of factored Fourier series by using absolute matrix summability method.

**Keywords:** Summability factors, absolute matrix summability, Fourier series, infinite series, Hölder's inequality, Minkowski's inequality.

**2010 Mathematics Subject Classification:** 26D15, 42A24, 40F05, 40G99.

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## A New Generalization on Absolute Riesz Summability

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In this paper, we have extended a theorem dealing with absolute Riesz summability.

**Keywords:** Summability factors, absolute matrix summability, infinite series, Hölder's inequality, Minkowski's inequality.

**2010 Mathematics Subject Classification:** 26D15, 42A24, 40F05, 40G99.

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## Absolute Matrix Summability on Quasi Power Increasing Sequences

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In this paper, we have generalized a new summability factor theorem for infinite series involving quasi power increasing sequences. Some new results are also deduced.

**Keywords:** Summability factors, absolute matrix summability, infinite series, Hölder's inequality, Minkowski's inequality.

**2010 Mathematics Subject Classification:** 26D15, 42A24, 40F05, 40G99.

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## Sufficient Tauberian Conditions for the $(\overline{N}, p)$ Summability of Sequences

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Let  $p = (p_n)$  be a sequence of nonnegative numbers satisfying

$$P_n := \sum_{k=0}^n p_k \rightarrow \infty, \quad n \rightarrow \infty. \quad (1)$$

The weighted mean of a sequence  $(u_n)$  is given by  $\sigma_{n,p}^{(1)}(u) := \frac{1}{P_n} \sum_{k=0}^n p_k u_k$  for  $n \in \mathbb{N}_0$ . A sequence  $(u_n)$  is called  $(\overline{N}, p)$  summable to  $s$  if  $\lim_{n \rightarrow \infty} \sigma_{n,p}^{(1)}(u) = s$ .

It is known that  $\lim_{n \rightarrow \infty} u_n = s$  implies  $\lim_{n \rightarrow \infty} \sigma_{n,p}^{(1)}(u) = s$  if and only if (1) is satisfied. However, the converse is not true in general. In this paper our aim is to find conditions under which the converse implication holds. Exactly, we prove under which conditions the convergence of a sequence follows from  $(\overline{N}, p)$  summability.

**Keywords:** Tauberian theorem,  $(\overline{N}, p)$  summability method, two-sided conditions.

**2010 Mathematics Subject Classification:** 40E05, 40G05.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## $p$ -Ward Continuity in 2-Normed Spaces

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In this paper, the concept of a quasi-Cauchy sequence is generalized to a concept of a  $p$ -quasi-Cauchy sequence for any fixed positive integer  $p$  in 2-normed space  $X$ . Some interesting theorems related to  $p$ -ward continuity and uniform continuity are obtained. A sequence  $(x_n)$  in a 2-normed space  $X$  is called  $p$ -quasi-Cauchy if  $\lim_{n \rightarrow \infty} \|x_{n+p} - x_n, z\| = 0$  for each  $z \in X$ . It turns out that if a function  $f$  defined on a subset of  $X$  is uniformly continuous then  $f$  preserves  $p$ -quasi-Cauchy sequences for all positive integer  $p$ .

**Keywords:** Sequences, series, summability, continuity, compactness, 2-normed spaces.

**2010 Mathematics Subject Classification:** 40A05, 40A30, 26A15, 42A65, 54C30.

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## 6.4 Fixed Point Theory

Session Organizer: Duran Turkoglu and Hakan Sahin

Fixed point theory has an important place in other branches as well as mathematics because of its applicability. Indeed, fixed point theory can be used to find solutions of nonlinear integral equations and solutions of first order differential equations. On the other hand, it might be used to determine the conditions for the existence of Nash equilibrium in strategic games too. Therefore, this topic has attracted attention of many scientist. The Fixed Point Theory session of the 3rd International Conference of Mathematical Sciences (ICMS 2019) provides a platform to researchers interested in this area to discuss and present of their ideas. In this session, there are 23 oral and 3 poster presentations. Topics of these presentations include nonlinear fractional differential equations, common fixed point, time scales, best approximation theory, almost picard operators, multi-fractal space but are not limited to. Besides, 7 extended abstracts will be published in AIP Conference Proceedings.

International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

## New Approach to Find Multi-Fractal Dimension of Multi- Fuzzy Fractal Attractor Sets Based on Iterated Function System

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In nature, the objects are not single fractal sets. They are collection of complex multiple fractals and this collection characterizes the multi-fractal space which is a generalization of the fractal space. While fractal space includes a fractal set, a multi-fractal space includes the union of fractals. So, the dimension of fractal sets leads to the dimension of multi-fractal sets. In this paper, we proposed a theoretical proof to define the multi-dimension of multi- fuzzy fractal attractor of  $n$  objects for the self-similar fractals sets  $A = \prod_{i=1}^n A_i = (A_1, A_2, \dots, A_n)$  of the contraction mapping  $W^{**} : \prod_{i=1}^n H(F(X_i)) \rightarrow \prod_{i=1}^n H(F(X_i))$  with contractivity factor  $r = \max\{r_i, i = 1, 2, \dots, n\}$  where  $H(F(X_i))$  is a fuzzy fractal space for each  $i = 1, 2, \dots, n$  over a complete metric space  $(\prod_{i=1}^n H(F(X_i)), D^*)$  where for all  $B_i$  belong to  $H(F(X_i))$ , there exists  $B^*$  belong to  $(\prod_{i=1}^n H(F(X_i)))$  such that  $W^{**}(B^* = \prod_{i=1}^n B_i) = \prod_{i=1}^n (\bigcup_{j=1}^n \bigcup_{k=1}^{k(i,j)} \omega_{ij}^{*k}(B_j) = \prod_{i=1}^n W_i(B^*))$ . By supposing that  $M(t) = \left( \sum_k (r_{ij}^{*k})^{FD} \right)_{n \times n}$  is the matrix associated with the the fuzzy contraction mapping  $\omega_{ij}^{*k}$  with contraction factor  $r_{ij}^{*k}, \forall i, j = 1, 2, \dots, n, \forall k = 1, 2, \dots, k(i, j)$ , for all  $t \geq 0$ , and  $h(t) = \det(M(t) - I)$ . Then we prove that if there exists  $FD$  such that  $h(FD) = 0$ , then  $FD$  is the multi fractal dimension to multi fuzzy-fractal sets of IFS and moreover  $M(FD)$  has a fixed point in  $R^n$ .

Keywords: Fractal space, multi-fractal space, multi-fuzzy fractal space, box-counting dimension, fractal dimension.

2010 Mathematics Subject Classification: 28A80, 37F05, 81Q35.

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## On the Study of Nonlinear Fractional Differential Equations on Unbounded Interval

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By the means of the variation of constants formula and some analytical skills, we use Banach contraction principle to investigate in this paper an uniqueness and existence of unbounded solution for nonlinear differential equations of fractional orders in weighted Banach space. At last, we present an illustrative example to support our main result.

**Keywords:** Fractional differential equations, Banach contraction principle, Existence and uniqueness.  
**2010 Mathematics Subject Classification:** 34A08, 47H10.

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## Chatterjea Type Fixed Point Results on Metric Spaces

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In this talk, we present some fixed point theorems for Chatterjea type mappings on metric space. First, we consider the Chatterjea type contractiveness of a self mapping on compact metric space and hence we give an Edelstain type fixed point theorem. Then we obtain a fixed point result for an involution of such type mappings with contractive factor  $K < 1$ . Finally, by taking into account approximation sequences we give another fixed point theorem.

Keywords: Fixed point, compact metric, Chatterjea type mapping.

2010 Mathematics Subject Classification: 54H25, 47H10.

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## Fixed Point Results with $\theta$ -Function on Quasi Metric Space

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In this presentation by considering the  $\theta$  function which is a recent technique for contractions on metric space, we introduce a new type contraction on quasi metric space. Then, we give some fixed point results for  $\alpha$ -admissible mappings satisfying this new contraction on different kind of complete quasi metric space.

Keywords: Fixed point, quasi metric space,  $\theta$ -contraction.  
2010 Mathematics Subject Classification: 54H25, 47H10.

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## Common Fixed Points for Two Mappings

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The aim of the present paper is to obtain a common fixed point theorem by employing the new introduced notion of weak subsequential continuity [1]. This new notion is a proper generalization of weak reciprocal continuity introduced recently by R.P. Pant et al. [3] and sequential continuity of type  $(\mathcal{A}_f)$  and  $(\mathcal{A}_g)$  introduced by D. Gopal et al. [2], and is applicable to compatible mappings as well as  $R$ -weakly commuting mappings of type  $(\mathcal{A}_f)$  (resp.  $(\mathcal{A}_g)$  and  $(\mathcal{P})$ ). Our results generalize several fixed point theorems.

**Keywords:** Reciprocal continuity, weak reciprocal continuity, sequential continuity of type  $(\mathcal{A}_f)$ , sequential continuity of type  $(\mathcal{A}_g)$ , subsequential continuity, weak subsequential continuity, compatible mappings, sub-compatible mappings,  $R$ -weakly commuting mappings of type  $(\mathcal{A}_f)$ ,  $R$ -weakly commuting mappings of type  $(\mathcal{A}_g)$ .

2010 Mathematics Subject Classification: 47H10, 37C25, 54H25, 55M20.

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## Complexity Analysis of Primal-Dual Interior Point Methods for Semidefinite Programming Based on a New Kernel Function with an Hyperbolic Barrier Term

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In this paper, we present a new primal-dual interior point algorithm for SDP problems based on a new kernel function. By simple analysis, we derive the iteration bounds  $\mathcal{O}\left(n^{\frac{3}{4}} \ln \frac{n}{\epsilon}\right)$  for large-update methods and  $\mathcal{O}\left(\sqrt{n} \ln \frac{n}{\epsilon}\right)$  for small-update methods. These results match the currently best known iteration bounds for large- and small-update methods based on the hyperbolic kernel functions.

**Theorem 1.** Let  $\Psi_0$  be an upper bound for  $\Psi(\beta V)$  during the process of the algorithm and let  $\tau \geq 1$ . Then, the total number of iterations to obtain an approximation solution with  $n\mu \leq \epsilon$  is bounded by

$$\left[ 96 \Psi_0^{\frac{3}{4}} \right] \left[ \frac{1}{\theta} \ln \frac{n}{\epsilon} \right].$$

Keywords: Semidefinite programming, primal-dual IPMs, complexity analysis.  
 2010 Mathematics Subject Classification: 90C22, 90C51, 90C31.

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## Almost Picard Operators

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The concept of Picard operator is one of the most important concept of fixed point theory. As known, a self mapping  $T$  of a metric space  $X$  is called Picard operator (PO) if it has unique fixed point and every Picard iteration sequence converges to this fixed point. There some weaker forms of PO in the literature as weakly Picard operator (WPO) and pseudo Picard operator (PPO). In this study, we present a new kind of PO as almost Picard operator (APO) and we show the differences from the others. Then we show that every continuous  $P$ -contractive self mapping of a compact metric space is APO. Also we present some open problems.

Keywords: Picard operator, fixed point, contractive mapping.

2010 Mathematics Subject Classification: 47H10, 54H25.

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## Nonconvex Second Order Sweeping Processes in Hilbert Spaces

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The aim of this work is to show how a fixed point approach can lead to the general existence theorem in the infinite dimensional space to the problem

$$\begin{cases} \ddot{u}(t) \in N_{C(t,u(t))}(\dot{u}(t)) + F(t, u(t), \dot{u}(t)) \text{ a.e on } [T_0, T], \\ \dot{u}(t) \in C(t, u(t)), \text{ for all } t \in [T_0, T], \\ u(0) = u_0, \dot{u}(0) = v_0 \in C(T_0, u_0), \end{cases} \quad (1)$$

where  $F$  is a Carathodory unbounded function and  $C(t, u)$  is uniformly prox regular set moving in an absolutely continuous way with respect to  $t$  and Lipschitz continuous with respect to the state  $u$ . Problem (1) includes as a special case the following evolution quasi-variational inequality:

$$\begin{aligned} \text{Find } u : [T_0, T] \rightarrow H, u(T_0) = u_0, \dot{u}(0) = v_0 \in C(T_0, u_0), \text{ such that } \dot{u}(t) \in C(t, u(t)) \text{ a.e. on } [T_0, T], \text{ and} \\ 0 \leq \langle \ddot{u}(t), w - \dot{u}(t) \rangle + a(u(t), w - \dot{u}(t)) \end{aligned} \quad (2)$$

for all  $w \in C(t, u(t))$ .

Keywords: Differential inclusion, prox-regular sets, normal cones.  
 2010 Mathematics Subject Classification: 49J52, 46N10, 58C20

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## A Logarithmic Barrier Method for Linear Programming using a New Minorant Function

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This paper presents a logarithmic barrier method without line search for solving linear programming problem. The descent direction is the classical Newton's one. However, the displacement step is determined by a simple and efficient technique based on the notion of the minorant function approximating the barrier function. We have obtained three minorant function:

- The first one is  $G_0(t) = \gamma_0 t - \ln(1 + \beta_0 t) - (n - 1) \ln(1 + \alpha_0 t)$ ,  $t \in [0, \hat{t}_0[$ , where  $\hat{t}_0 = \sup\{t, 1 + \alpha_0 t > 0\}$ .
- The second is defined by  $G_1(t) = \hat{\gamma} t - \hat{\sigma} \ln(1 + \hat{\beta} t)$ ,  $t \in [0, \hat{t}_1[$ , with  $\hat{t}_1 = \sup\{t, 1 + \hat{\beta} t > 0\}$ .

Where  $\hat{\beta} = \beta_0 = \bar{z} + \sigma_z \sqrt{n - 1}$ ,  $\hat{\sigma} = \frac{\|z\|^2}{\hat{\beta}^2}$  and  $\hat{\gamma} = \hat{\sigma} \hat{\beta} - \|z\|^2$ .

- The last function is defined by  $G_{eff}(t) = \gamma_0 t - p \ln(1 + t\alpha_0)$ ,  $t \in [0, \hat{t}_0[$ ,  $p \leq 0$ .

Keywords: Linear Programming, logarithmic barrier methods, minorant function.

2010 Mathematics Subject Classification: 90C22, 90C51.

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## Existence and Uniqueness of Solution for the Problem in the Times Scales

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The purpose of this work is the construction of minimal and maximal solutions for a class of second order quasilinear elliptic subject to nonlocal boundary conditions. More specifically, we consider the following nonlinear boundary value problem

$$(P) \begin{cases} -(\varphi_p(u^\Delta))^\Delta = f(x, u^\alpha) & \text{in } (a, b)_T, \\ u(a) - a_0 u^\Delta(a) = g_0(u), \\ u(\sigma(b)) + a_1 u^\Delta(\sigma(b)) = g_1(u) \end{cases}$$

where  $p > 1$ ,  $(\varphi_p(u)) = |u^{p-2}u|$ ,  $(\varphi_p(u^\Delta))$  is the one-dimensional p-laplacien,  $f : [a, b]_T \times \mathbb{R} \rightarrow \mathbb{R}$  is rd-continuous function,  $\alpha = \rho$  or  $\alpha = \sigma$ ,  $g_i : C_{rd}([a, b]_T) \times C_{rd}([a, b]_T) \rightarrow \mathbb{R}$  ( $i = 0, 1$ ) are rd-continuous and  $a_0, a_1$  are positive real numbers.

**Keywords:** Quasilinear elliptic equation, time scale, nonlocal boundary conditions, upper and lower solutions, monotone and iterative technique.

**2010 Mathematics Subject Classification:** 34B15, 39A10.

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## Fixed Point Approach for Differential Inclusions Governed by Subdifferential Operators

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In this work, we deal with a first order differential inclusion governed by a subdifferential operator and containing a set-valued perturbation. The existence of absolutely continuous solution is stated by applying a weak version of Kakutani's fixed point Theorem.

Keywords: Differential inclusion, subdifferential operator, perturbation

2010 Mathematics Subject Classification: 34A60; 49J53.

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## Some Fixed Point Results on Modular $\mathcal{F}$ -Metric Spaces

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Jleli and Samet(2018)[8] introduced a new concept, named a  $\mathcal{F}$ -metric space, as a generalization of the notion of metric space. In this paper, we compare the topology produced by modular metric and by modular  $\mathcal{F}$ -metric and cover some useful properties of this topology for fixed point theorems in future studies. Then, we prove Banach contraction principle in modular  $\mathcal{F}$ -metric spaces.

Keywords:  $\mathcal{F}$ -metric space, fixed-point theorems, modular metric spaces.

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## Three-Step Projective Methods for Solving the Split Feasibility Problems

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In this work, we focus on studying the method for solving the split feasibility problem (SFP) in Hilbert spaces. Based on the CQ-algorithm and the self-adaptive technique, we introduce new three-step iterative schemes for solving the split feasibility problem. Then both weak and strong convergence theorems are proved under some mild conditions. Numerical examples are given to show its efficiency in signal processing. Some comparison to various methods are also provided in this paper.

**Keywords:** Self-adaptive method, split feasibility problem, convergence theorems, Hilbert space, CQ-algorithm  
**2010 Mathematics Subject Classification:** 47H10, 54H25.

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## A Fixed Point Approach for a Differential Inclusion Governed by the Subdifferential of PLN Functions

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In this paper, we apply the Kakutani's fixed point theorem to a first order differential inclusion governed by the subdifferential of primal lower nice functions in a Hilbert space.

Keywords: differential inclusions, subdifferentials, primal lower nice functions.  
2010 Mathematics Subject Classification: 49A52, 49J53, 34A60.

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## Fixed Point Theorems on Orthogonal Metric Spaces via Altering Distance Functions

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In 1984, Khan et al. [3] established some fixed point theorems in complete and compact metric spaces by using altering distance functions. In 2017, Gordji et al. [2] described the notion of orthogonal set and orthogonal metric spaces. In their work, they held an generalization of Banach fixed point theorem in this interesting defined construction and in addition, applied their acquired results to demonstrate the existence of a solution of an ordinary differential equation. In this paper, some fixed point theorems on orthogonal metric spaces via altering distance functions are presented inspired by [3] and [2].

Keywords: altering distance function, fixed point, orthogonal metric space.

2010 Mathematics Subject Classification: 47H10, 54H25.

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## Fixed Point Theory on Spaces with Vector-Valued Metrics

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In this work, we prove some common fixed point theorems for two operators on a set endowed with one or two vector-valued metrics. The use of vector-valued metrics makes it possible for each equation of a system to have its own Lipschitz property, while the use of two such metrics makes it possible for the Lipschitz condition to be expressed with respect to an incomplete metric.

**Definition 1.** Let  $X$  be a nonempty set. A mapping  $d : X \times X \rightarrow \mathbb{R}_+^n$  is called a vector-valued metric on  $X$  if the following properties are satisfied:

1.  $d(x, y) = 0$  if and only if  $x = y$  for all  $x, y \in X$ ;
2.  $d(x, y) = d(y, x)$  for all  $x, y \in X$ ;
3.  $d(x, z) \leq d(x, y) + d(y, z)$  for all  $x, y, z \in X$ .

A set  $X$  equipped with a vector-valued metric  $d$  is called a generalized metric space.

**Keywords:** Fixed point, vector-valued metric, matrix convergent to zero.

**2010 Mathematics Subject Classification:** 37H10, 54H25.

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## Homoclinic and Heteroclinic Bifurcations for Henon Like Diffeomorphism

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This work describe the existence of Henon attractor for diffeomorphism emerged of an endomorphism of one dimension. We determine the properties of attraction basins, stable and unstable manifolds, homoclinic and heteroclinic points and the role of this points to the global bifurcation of attraction basin. We study this diffeomorphism in dependance of at least three parameters and uncover many fascinating dynamical characteristics, using both analytic perturbation theory and numerical methods.

**Definition 1.** Let  $S$  be a saddle fixed point of  $T$  ; a point  $q$  is called homoclinic to  $S$ , if  $q \in W^s(S) \cap W^u(S)$  and  $q \neq S$ .  $q$  is a transversal homoclinic point, so  $W^s(S)$  intersects transversely  $W^u(S)$ .

Keywords: Attractor, bifurcation, fixed point.

2010 Mathematics Subject Classification: 37G10, 37J20, 37J15.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Convergence Theorems for Three G-Nonexpansive Mappings in Hilbert Spaces with Graphs by Modifying SP and Noor Iterations with Shrinking Projection Methods

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In this work, we propose two new iterative schemes by modifying the shrinking projection method with Noor and SP iterations. The strong convergence theorems are given for obtaining a common fixed point of three G-nonexpansive mappings in a Hilbert space with a directed graph under some suitable conditions. Finally, we give some numerical examples for supporting our main theorems and compare the rate of convergence of some examples under the same conditions.

Keywords: SP-iteration, shrinking projection, G-nonexpansive mappings, Noor-iteration, Hilbert space.  
2010 Mathematics Subject Classification: 47H10, 54H25.

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## Existence Results of First-Order Differential Equations without Integral Boundary Conditions at Resonance

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This work is concerned with the construction of solutions for a first order differential equation with integral boundary condition at resonance such that

$$\begin{cases} x'(t) = f(t, x(t)), t \in J = [0, T], \\ x(0) = \int_0^T g(s)x(s)ds, \end{cases}$$

where  $f : J \times \mathbb{R} \rightarrow \mathbb{R}$  and  $g : J \rightarrow \mathbb{R}$  are continuous functions and  $T > 0$ . We also give some examples to illustrate our results.

**Keywords:** First order differential equations, upper and lower solutions, monotone iterative technique, resonance.

**2010 Mathematics Subject Classification:** 34B10, 34B15

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## 6.5 Numerical Functional Analysis

Session Organizer: Allaberen Ashyralyev

This section of abstract book is a collection of abstracts of 42 original papers by 80 authors. These papers are presented at the Special Session "Numerical Functional Analysis" which will be held as a part of the International Conference of Mathematical Sciences (ICMS 2019) 04 September - 08 September 2019 in Maltepe University, Istanbul, Turkey.

The main aim of this special session is to provide impetus, motivation and to bring together researchers and scientists working in the fields of Numerical Functional Analysis and Operator Theory and Applications by providing a forum for the academic exchange of ideas and recent research works.

They are include different problems of well-posedness of differential and difference problems, construction and investigation of analytic and numerical methods for ordinary and partial differential equations and dynamic equations, investigation of stochastic equations, study of identification problems, fractional calculus, and mathematical modelling.

The selected papers examine wide ranging and cutting edge developments in various areas of Numerical Functional Analysis and Operator Theory and their applications. The papers give a taste of current research. We feel the variety of topics will be of interest to both grade students and researchers.

Further, we are very grateful to all scientists from Turkey, Algeria, Kazakhstan, Kyrgyzstan, Morocco, Russia, Iraq, India, Bulgaria, Libiya, Turkmenistan and Nijeria for their interest in Special Session "Numerical Functional Analysis" as a part of ICMS 2019.

International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

## Solvability of a Mixed Problem for a Heat Equation with an Involution Perturbation

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The theory and applications of ordinary differential equations with involutions have been investigated widely in the literature. In the present work, we study the partial differential equations with involutions. The mixed problems for the parabolic type equation, with constant and variable constants, corresponding to the Neumann type boundary conditions is investigated. Sufficient conditions for the initial data are found when the problem under study has a unique solution. The representation of the solution in the form of partial sums of the Fourier series in eigenfunctions is found.

### Acknowledgment

The works was supported by the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan, grant AP 05131225.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

## Mixed Problem for a Wave Equation with an Involution Perturbation

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In the present work, we study the partial differential equations with involutions. The mixed problems for the hyperbolic type equation, with constant and variable constants, corresponding to the Dirichlet type boundary conditions is investigated. Sufficient conditions for the initial data are found when the problem under study has a unique solution. The representation of the solution in the form of partial sums of the Fourier series in eigenfunctions is found.

### Acknowledgment

The works was supported by the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan, grant AP 05131225.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Numerical Analysis of the Influence of the Initial Parameters on the Convergence Rate of the Approximate Solution of the Boundary Value Problem

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In the present paper, the boundary value problem is considered of a controlled thermal process with an Fredholm integral operator in the case when the action of external heat flow occurs in the form of several moving point sources. In mathematical formalization, the generalized Dirac delta function was used. The procedure are briefly outlined for constructing a generalized solution of the boundary value problem and its approximations. It is shown that the presence of an integral operator determines the construction of two types of approximations and their convergence is proved. Sufficient conditions are found for convergence of approximations to a point solution. The main attention was paid to the influence of the parametric variation of the problem on the convergence rate of the approximations. The numerical calculations is given in tabular form.

Keywords: Boundary value problem, generalized solution, approximation.

2010 Mathematics Subject Classification: 46N10.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

## A Crank Nicolson Difference Scheme for the System of Nonlinear Observing Epidemic Models

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Classical epidemic SIR, SIS, and SEIR models have been proposed and studied by many authors in (see, e.g., [1]- [2] and the references given therein). Theorems on existence and uniqueness of the bounded solution of linear and nonlinear systems are established in ([3],[4]). In the present paper, a Crank Nicolson difference scheme and iteration method are used for the approximate solution of system of nonlinear observing epidemic model. Numerical results are provided.

**Keywords:** Epidemic models, Crank-Nicholson difference scheme, realization in computer

**2010 Mathematics Subject Classification:** 35K40, 65M12, 92B05

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

## A Numerical Algorithm for the Source Identification Parabolic-Elliptic Problem

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The theory and applications of various local and nonlocal boundary value problems for parabolic-elliptic equations have been investigated by many scientists in (see, e.g., [1]- [4] and the references given therein). In the present paper, a space-dependent identification problem for parabolic-elliptic equations is studied. The first and second order of accuracy difference schemes for the numerical solution of the source identification parabolic-elliptic problems are presented. Numerical results are given.

**Keywords:** Source identification problem, parabolic-elliptic, difference schemes, numerical experience.

**2010 Mathematics Subject Classification:** 35J25, 47E05, 34B27.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

## A Numerical Algorithm for the Source Identification Elliptic-Telegraph Problem

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The theory of local and nonlocal boundary value problems for elliptic-telegraph equations and its applications have been investigated by many scientists in [1]-[4]. In the present study, a space-dependent identification problem for elliptic-telegraph equations is studied. Stability estimates for the solution of the source identification problem are established. Furthermore, the first and second order of accuracy difference schemes for the numerical solution of the source identification one dimensional elliptic-hyperbolic equations problems are presented. Some numerical results are presented.

**Keywords:** Source identification problem, elliptic-telegraph, difference schemes, numerical experience.

**2010 Mathematics Subject Classification:** 35J25, 47E05, 34B27.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

## A Numerical Algorithm for the Involutory Parabolic Problem

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In this study, initial value problem for the involutory parabolic equation

$$\frac{du(t)}{dt} + aAu(t) + bAu(d-t) = f(t), t \in (-\infty, \infty), u\left(\frac{d}{2}\right) = \varphi \quad (1)$$

in a Hilbert space  $H$  with the self-adjoint positive definite operator  $A$  is investigated. Here,  $a, b, d \in (-\infty, \infty)$ ,  $f(t)$  is the continuous and bounded abstract function defined on  $(-\infty, \infty)$  with values in  $H$ .

Applying Fourier series, Laplace and Fourier transform methods, we can obtain the solution of several parabolic type involutory differential problem. Furthermore, the first and second order of accuracy difference schemes for the numerical solution of the initial boundary value problem for one dimensional parabolic type involutory partial differential equation are presented. Numerical results are given.

**Keywords:** Hilbert space, involutory parabolic equation, difference schemes, numerical experience.

**2010 Mathematics Subject Classification:** 35J25, 47E05, 34B27.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

## Finite Difference Method for the Third-Order Partial Differential Equation with Nonlocal Boundary Conditions

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The theory and applications of local and nonlocal boundary value problems for a third-order partial differential equations have been investigated widely in the literature. In the present work, we study the nonlocal boundary value problem

$$\left\{ \begin{array}{l} \frac{d^3 u(t)}{dt^3} + \mu \frac{d^2 u(t)}{dt^2} + A \frac{du(t)}{dt} = f(t), \quad 0 < t < 1, \mu > 0, \\ u(0) = \gamma u(\lambda) + \varphi, \quad u'(0) = \alpha u'(\lambda) + \psi, |\gamma| < 1, \\ u''(0) = \beta u''(\lambda) + \xi, \quad |1 + \beta\alpha| > |\alpha + \beta|, 0 < \lambda \leq 1 \end{array} \right.$$

for third order partial differential equations in a Hilbert space  $H$  with a self-adjoint positive definite operator  $A$ . The main theorem on stability of this problem is established. The stability estimates for the solution of three problems for partial differential equations are obtained. Three-step difference schemes for the approximate solution of nonlocal boundary-value problem for the third-order partial differential equation are presented. Numerical experiments results are provided.

**Keywords:** Nonlocal boundary-value problem, third-order partial differential equation, difference schemes, numerical experience.

2010 Mathematics Subject Classification: 35J25, 47E05, 34B27.

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## Time-Dependent Source Identification Problem for the Schrödinger Equation with Nonlocal Boundary Conditions

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**Abstract.** In the present study, a source identification problem for a one-dimensional Schrödinger equation with nonlocal boundary conditions is investigated. Stability estimates for the solution of the source identification problem are established. A first order of accuracy difference scheme for the numerical solution of this problem is presented. Stability estimates for the solution of this difference scheme are established. Numerical results are provided.

**Keywords:** Source identification problem, Schrödinger differential equations, difference scheme.

2010 Mathematics Subject Classification: 35J25, 47E05, 34B27.

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04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

## A Numerical Algorithm for the Third Order Partial Differential Equation with Time Delay

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The theory and applications of various local and nonlocal boundary value problems for the third order partial differential equations have been investigated by many scientists in (see, e.g., [1]- [3] and the references given therein). In the present paper, the initial value problem for the third order partial differential equations with time delay is studied. The first and second order of accuracy difference schemes for the numerical solution of the third order partial differential equations with time delay are presented. The illustrative numerical results are provided.

**Keywords:** Third order pde, time delay, difference schemes

**2010 Mathematics Subject Classification:** 35G10, 65M12, 34D20

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## A Numerical Algorithm for the Involutory Schrödinger Type Problem

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It is known that various problems in physics lead to the Schrödinger equation. Methods of solutions of the problems for Schrödinger equation have been studied extensively by many researchers (see, e.g., [1, 2, 3, 4], and the references given therein).

In this study, applying Fourier series, Laplace and Fourier transform methods, we can obtain the solution of several Schrödinger type involutory problem. The first and second order of accuracy difference schemes for the numerical solution of the initial boundary value problem for one dimensional Schrödinger type involutory equation are presented. Numerical results are given.

**Keywords:** Difference schemes, involutory, Schrödinger type equation.

**2010 Mathematics Subject Classification:** 35J25, 47E05, 34B27.

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04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

## Numerical solution of the system of fractional differential equations observing epidemic models

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Classical epidemic fractional and differential models have been proposed and studied by many authors in (see, e.g., [1]- [4]). In the present study, numerical solutions of fractional differential equations observing epidemic model problems are investigated. First and second order of accuracy difference schemes are presented for the solution of the one dimensional epidemic problems and the numerical procedure for implementation of these schemes is discussed.

**Keywords:** Epidemic models, fractional differential equations, difference scheme, realization in computer

**2010 Mathematics Subject Classification:** 35K40, 34A08, 92B05

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04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

## Well-posedness of source identification problem for the elliptic equation in a Banach space

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**Abstract.** In the present paper, the source identification elliptic problem

$$\begin{cases} -u''(t) + Au(t) = f(t) + p, & 0 < t < T, \\ u'(0) = \varphi, u'(T) = \psi, u(\gamma) = \zeta, & 0 \leq \gamma \leq T \end{cases} \quad (1)$$

with a positive operator  $A$  in an arbitrary Banach space  $E$  is studied. Here smooth function  $f : [0, T] \rightarrow E$  and elements  $\varphi, \psi, \zeta$  are given.

The exact estimates for solution of identification problem are established in Hölder norms. In applications, coercive stability estimates for the solution of three elliptic boundary value problems are obtained.

**Keywords:** Elliptic equations, source identification, positivity, exact estimates, coercive stability.

**2010 Mathematics Subject Classification:** 35N25, 35J67.

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## On elliptic differential and difference problems in a Hilbert space with special type nonlocal conditions

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The nonlocal conditions  $v(0) = v(2\pi)$ ,  $\int_0^{2\pi} v(s)ds = 0$  were considered by Sobolevskii in [1] for the study of summability of Fourier series in  $C$ -norm. Such type conditions are useful for study of several differential and difference problems with such type nonlocal conditions. In the present paper elliptic differential and difference problems

$$-u''(t) + Au(t) = f(t), 0 < t < T, u(0) = u(T), \int_0^T u(s)ds = 0,$$

$$-\frac{u_{k+1} - 2u_k + u_{k-1}}{\tau^2} + Au_k = f(t_k), 1 \leq k \leq N-1, N\tau = T, u_0 = u_N, \sum_{k=1}^N u_k = 0$$

in a Hilbert space  $H$  with the positive definite operator  $A \geq \delta I, \delta > 0$  are investigated. The stability estimates in max norm in one variable for the solutions of these problems are established. Applications are presented for several elliptic differential and difference problems with such type nonlocal conditions.

**Keywords:** Elliptic equation, positive operators, Hilbert space, stability, Sobolevskii type nonlocal conditions.

2010 Mathematics Subject Classification: 35J25, 47E05, 34B27.

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04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

## Optimal Control Approach to Study Two Strain Malaria Model

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Control of Malaria is very difficult due to anti malarial drug resistant diseases. Many control measures exist such as; insecticides treated bed net (ITNs) and drug treatments. Most mathematical models in literature used constant control measures which is not realistic. Here we use optimal control as a measure in curtailing the disease spread. The control function is added in the sensitive strain. Analysis of the controller was carried out.

**Keywords:**Optimal control, mathematical model, Hamiltonian, two strain, malaria

**2010 Mathematics Subject Classification:** 93C10, 93C40

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## On a Boundary Problem for a Nonlocal Poisson Equation with Boundary Operators of the Hadamard Type

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In this paper the solvability problems of some boundary value problems for a nonlocal Poisson equation are studied. A non-local Poisson equation is represented by using some orthogonal matrix. The properties and examples of such matrices are given. In the current boundary value problem, which being considered in the paper, the fractional order differentiation operators are used as boundary operators. These operators are defined as derivatives of the Hadamard-Caputo type. Note that in particular cases of the parameters of the boundary conditions we obtain well known conditions of the Dirichlet, Neumann, and Robin type problems [1]. For the problems under consideration, theorems on the existence and uniqueness of solutions are proved. The exact solvability conditions for the problem under study are found. In addition, we obtained representation for the solution of the fractional boundary problem for Poisson equation.

**Keywords:** Boundary value problems, fractional derivatives, Existence and uniqueness, nonlocal equation, Poisson equation.

**2010 Mathematics Subject Classification:** 31A25,35J05,35J25.

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## On the Numerical Analysis of a Nonlinear Fractional Schrödinger Equation with Neumann Boundary Condition

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In the present paper, a nonlinear fractional Schrödinger equation is investigated with a numerical approach. First and second orders of accuracy difference schemes are constructed for a mixed problem for a nonlinear fractional Schrödinger equation with Neumann boundary condition. Results of numerical experiments support the convergence of solutions of constructed first and second order of accuracy difference schemes to exact solution of the problem.

Keywords: Fractional derivative, convergence, Neumann boundary condition.

2010 Mathematics Subject Classification: 65J08, 65J15, 35Q55.

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## An Iterative Regularization Method for an Ill-Posed Bi-Fractional Problem

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In this talk, we are concerned with the problem of approximating a solution of an ill-posed bi-fractional problem. In order to overcome the instability of the original problem, we propose a regularizing strategy based on the Kozlov-Mazya iteration method. Finally, some other convergence results are also established.

Keywords: Fractional problem, inverse problem, iterative regularization method.

2010 Mathematics Subject Classification: 35R25, 47A52.

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## Numerical Solution of Neumann Type Elliptic Overdetermined Multipoint Mixed Boundary Value Problem

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The theory and applications of various source identification problems for elliptic equations have been studied in [1]- [4] (see the references given therein).

In the present paper, Neumann type elliptic overdetermined multipoint boundary problem is discussed. The first and second order of accuracy difference schemes for the numerical solution of elliptic overdetermined multipoint boundary value problems are presented. Stability estimates for solutions of difference schemes are obtained. Numerical results for mixed boundary value problem are given.

**Keywords:** Inverse elliptic problem, difference schemes, numerical experience.

**2010 Mathematics Subject Classification:** 65J22, 39A14, 39A30.

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## A New Coupling Method for the Stokes-Darcy Model

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Inspired by the Galerkin and the particule method, a new approximation approach is recalled in this paper. We are interested specially to apply this method on the coupled Stokes-Darcy problem, with Beaver-Joseph and Saffman interface condition. Before studying the coupled problem, we present our new scheme for the Stokes problem lid a driven cavity, and after, we explicit our approach for the Stokes-Darcy model. In this work, we reduce the number of iterations in a very fast way. Theoretical analysis and numerical experiments shows the efficiency of our scheme.

Keywords: PDEs,  $\delta$ -Ziti method, Stokes-Darcy model.

2010 Mathematics Subject Classification: 34A45, 37N30, 46N30.

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04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

## Sinc Approximation of Solution of Integro-Differential Equation

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Many mathematical models of complex processes may be posed as integro-differential equations. Different numerical methods have been proposed and developed in recent years, such as quadrature method, collocation method, Galerkin method, expansion method and product-integration method.

The application of the Sinc-Galerkin method to an approximate solution of integro-differential equations were discussed in this study. The method is based on approximating functions and their derivatives by using the Whittaker cardinal function in order to determine the approximate solutions.

Without any numerical integration, the partial differential equation transformed to an algebraic equation system. For the numerical calculations, Maple is used and the results determined from the method are compared with the exact solutions. The results are illustrated both in table and graphically to show the rapid convergence and exceptional accuracy of the method.

**Keywords:** Sinc-Galerkin methods, Whittaker cardinal function, conform application, integro-differential equations.

**2010 Mathematics Subject Classification:** 35G15, 47E05, 47A62.

## Differential and Difference Variants of 2-d Nonlocal Boundary Value Problem with Poisson's Operator

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Applying [1] and [2] in rectangular  $\Pi = (0 < x < 1) \times (0 < y < \pi)$  in differential and difference variants of two-dimensional nonlocal boundary value problems with Poisson's operator are investigated.

$$\left\{ \begin{array}{l} \Delta u = f(x, y), (x, y) \in \Pi; \quad u(x, 0) = u(x, \pi) = 0, \quad 0 \leq x < 1; \\ u(0, y) = 0, \quad u(1, y) = \sum_{r=1}^n \alpha_r u(\zeta_r, y) - \sum_{s=1}^m \beta_s u(\eta_s, y), \quad 0 \leq y \leq \pi \\ 0 < \zeta_1 < \dots < \zeta_n < 1; \quad 0 < \eta_1 < \dots < \eta_m < 1; \quad \zeta_n < \eta_1; \quad \alpha_r > 0, \quad r = \overline{1, n}; \\ \beta_s > 0, \quad s = \overline{1, m}; \quad -\infty < \sum_{r=1}^n \alpha_r - \sum_{s=1}^m \beta_s \leq 1, \\ \Lambda Y = Y_{xx} + Y_{yy} = f(x, y), \quad (x_i, y_j) \in \Pi; \quad Y|_{y=0} = Y|_{y=\pi}, \quad 0 \leq x_i < 1; \\ Y|_{x=0} = 0; \quad LY = \sum_{r=1}^n \alpha_r \left\{ Y_{i_{\zeta_r}, j}^{\frac{[(i_{\zeta_r}+1)h_1 - \zeta_r]}{h_1}} + Y_{i_{\zeta_r}+1, j}^{\frac{[\zeta_r - i_{\zeta_r} h_1]}{h_1}} \right\} \\ - \sum_{s=1}^m \beta_s \left\{ Y_{i_{\eta_s}, j}^{\frac{[(i_{\eta_s}+1)h_1 - \eta_s]}{h_1}} + Y_{i_{\eta_s}+1, j}^{\frac{[\eta_s - i_{\eta_s} h_1]}{h_1}} \right\} - Y_{N_1, j} = 0, \quad j = \overline{1, N_2}; \\ 2h_1 < \min\{\zeta_1; \zeta_r - \zeta_{r-1}, r = \overline{2, n}; \eta_1 - \zeta_n; \eta_s - \eta_{s-1}, s = \overline{2, m}; 1 - \eta_m\}; \\ i_{\zeta_r} h_1 \leq \zeta_r < (i_{\zeta_r} + 1)h_1, \quad r = \overline{1, n}; \quad i_{\eta_s} h_1 \leq \eta_s < (i_{\eta_s} + 1)h_1, \quad s = \overline{1, m}; \\ h_1 = 1/N_1, \quad h_2 = \pi/N_2, \quad h_1 \leq C_0 h_2, \quad C_0 = const, \quad 0 \leq y_j \leq \pi. \end{array} \right.$$

**Theorem 1.** Let  $f(x, y) \in C^0(\overline{\Pi})$ , then classical solution of the differential problem exists and a priori estimate holds  $\|u(x, y)\|_{W_2^2(\Pi)} \leq C_1 \|f(x, y)\|_{L_2(\Pi)}$ .

**Theorem 2.** Let  $u(x, y) \in C^{(4)}(\overline{\Pi})$  for the solution of the differential problem, then solution  $Y$  of difference problem approximates classical solution  $u(x, y)$  with second order of accuracy in  $h = (h_1^2 + h_2^2)^{1/2}$  when  $h_2 \rightarrow 0$ .

Keywords: Poisson's operator, nonlocal boundary, difference problem.

2010 Mathematics Subject Classification: 34B10, 35J05, 65N06.

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04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

## Posterior Analysis of Weighted Erlang Distribution

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Erlang distribution is continuous probability distribution that has application in several field such as stochastic process and mathematical biology, due to its relation with exponential and gamma distribution. In the sense that, the duration of the successive calls follows the Erlang distribution, if individual telephone calls is exponentially distributed to the time period. In this study, Bayesian estimation is employed in the estimation of scale parameter od weighted Erlang distribution. The posterior distribution is derived under two informative priors, which are inverse exponential and inverse chi square prior. The Bayes estimated and their relative posterior risks are derived under the assumption of squared error loss function, and precautionary loss function. A Monte Carlo simulation is carried out in order to obtain the numerical value of the estimates. It was observed that squared error loss function performs best when inverse exponential prior is used.

**Keywords:** Erlang distribution, Bayesian estimation, loss function.

**2010 Mathematics Subject Classification:** 60E05, 62P20, 62E15.

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## Memory Efficient Algorithm for Solving the Inverse Gravimetry Problem of Finding Several Boundary Surfaces in Multilayered Medium

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For solving the inverse gravimetry problem of finding several boundary surfaces in a multilayered medium, the parallel algorithm was constructed and implemented for multicore CPU using OpenMP technology. The algorithm is based on the modified nonlinear conjugate gradient method with weighting factors previously proposed by authors. To reduce the memory requirements and computation time, the modification was constructed on the basis of utilizing the Toeplitz-block-Toeplitz structure of the Jacobian matrix of the integral operator.

The model problem of reconstructing three surfaces using the quasi-real gravitational data was solved on a large grid. It was shown that the proposed implementation reduces the computation time by 80% in comparison with the earlier algorithm based on calculating the entire matrix. The parallel algorithm shows good scaling of 94% on 8-core processor.

This work was financially supported by the Ministry of Education and Science of the Republic of Kazakhstan (project AP 05133873).

**Keywords:** Gravimetry problem, nonlinear gradient methods, parallel algorithms.

**2010 Mathematics Subject Classification:** 65M32, 65N21.

## Numerical Analysis of Convergence Rate of Approximation Solutions of a Boundary Value Problem for Oscillation Processes

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In the paper [1] the boundary problem was investigated for the controlled processes described by integro-differential equation of hyperbolic-type with Fredholm integral operator. A generalized solution and its approximations were constructed for the boundary value problem with boundary conditions of the second and third types. The convergence of approximations to the generalized solution was proved in the norm of Hilbert space.

In the present paper, the dynamics of convergence rate is investigated of the approximations depending on the changes of the stiffness coefficient of the elastic fixation. The results of the numerical analysis show that with increasing of stiffness coefficient (parameter  $\alpha$ ) of the elastic fixation the radius of convergence of Neumann series increases, and the convergence rate of the approximations to the exact solution accelerates.

Keywords: Boundary value problem, coefficient of stiffness, convergence.

2010 Mathematics Subject Classification: 46N10.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Solving Nonlinear Volterra-Fredholm Integro-Differential Equations Using He's Variational Iteration Method

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In this work, a nonlinear Volterra-Fredholm integro-differential equation is solved using He's variational iteration method. The approximate solution of this equation is calculated in the form of a sequence where its components are computed easily. Some examples are given to illustrate the effectiveness of the method. The results show that the method provides a straightforward and powerful mathematical tool for solving various a nonlinear Volterra-Fredholm integro-differential equations.

**Keywords:** Volterra-Fredholm integro-differential equations, He's variational iteration method, Convergence.  
**2010 Mathematics Subject Classification:** 65R10, 65K10, 65L20.

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04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

## The Effect of Harvesting Policy on an Eco-Epidemiological model

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Considering to the high extent spread of epidemics in the general of individuals of the miserable communities that causes of extinction, we suggested and investigated an eco-epidemiological system consisting two different (SIS and SI) infectious diseases in prey population, which are transmitted within prey species by the contact. The selective optimal harvesting on predator species is proposed and analyzed. The mathematical model involves four first-order nonlinear ordinary differential equations, which characterized the interaction among the susceptible prey, infected prey, and predator. The predator feeding depends on predation prey according to Holling type-II functional response, as well as, the nonlinear incidence for describing the transition of infectious diseases is used. The existence of bionomic equilibria, uniqueness, boundedness of the system solution and the optimal proportionate harvesting policy are discussed.

Keywords: Eco-epidemiological model, (SIS; SI), epidemic diseases, optimal harvesting effort, nonlinear incidence.

2010 Mathematics Subject Classification: 37B25.

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## Interpolation of Scattered Data in $\mathbb{R}^3$ Using Minimum $L_p$ -Norm Networks, $1 < p < \infty$

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We consider the extremal problem of interpolation of scattered data in  $\mathbb{R}^3$  by smooth curve networks with minimal  $L_p$ -norm of the second derivative for  $1 < p < \infty$ . The problem for  $p = 2$  was set and solved by Nielson [1]. Andersson et al. [2] gave a new proof of Nielson's result by using a different approach. It allowed them to set and solve the constrained extremal problem of interpolation of convex scattered data in  $\mathbb{R}^3$  by minimum  $L_2$ -norm networks that are convex along the edges of an associated triangulation. Partial results for the unconstrained and the constrained problems were announced without proof in [3]. Here we present complete characterization of the solutions to both the unconstrained and the constrained problems for  $1 < p < \infty$ .

Keywords: Extremal scattered data interpolation, minimum norm networks.

2010 Mathematics Subject Classification: 65D05, 65D17, 46N40.

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### On Unique Solvability of Nonlinear Coupled Systems

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In this study, the unique solvability and numerical solution of coupled sine Gordon equations is considered. The variational method is implemented to obtain unique weak solution. A new hybrid numerical method which combines finite difference method and fixed point iteration is used to solve the generalized form of coupled sine-Gordon equations. The novel first order of accuracy stable difference scheme is employed with a fixed point iteration for the approximate solution of the nonlinear coupled system. Numerical experiments are presented with error analysis, to verify the accuracy of the proposed method.

**Keywords:** Existence-uniqueness, finite difference method, fixed point theory.

**2010 Mathematics Subject Classification:** 35A01, 65M06, 37C25.

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## Construction of Unconditionally Stable Difference Schemes Based on Stability of Perturbed Difference Scheme

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In the paper we consider stability of an abstract three-layer “perturbed” difference scheme. The research method is based on obtaining discrete analogs of a priori weight Carleman type estimates [1, 2]. On the basis of stability of the “perturbed” scheme, unconditionally stable difference schemes have been constructed that approximate the ill-posed Cauchy problem, which is associated with the one-dimensional coefficient inverse problem [3].

This research was supported by the Ministry of Education and Science of the Republic of Kazakhstan (Project AP05133873).

**Keywords:** Difference scheme, ill-posed problem, perturbed scheme, stability.

**2010 Mathematics Subject Classification:** 65M32, 65N21.

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## An Inverse Diffusion-Wave Problem Defined in Heterogeneous Medium with Additional Boundary Measurement

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This talk deals an inverse problem to determine a space-dependent coefficient in a one-dimensional time fractional diffusion-wave equation defined in heterogeneous medium with additional boundary measurement. We construct the finite difference scheme for the direct problem based on the equivalent partial integro-differential equation. Under the weak smoothness conditions, we prove that our scheme is stable and convergent using the matrix analysis. Based on the least squares method with Tikhonov regularization is introduced to determine the space-dependent coefficient, and an inversion algorithm is performed by two numerical examples. This inversion algorithm is effective at least for this inverse problem.

**Keywords:** Inverse problem, time-fractional diffusion-wave equation, finite difference method.

**2010 Mathematics Subject Classification:** 65M32, 35R11, 65M06.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Symmetry Analysis of the Discrete Nonlinear Boundary Value Problems for the Wave Equation

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The invariance conditions for the boundary value problem of the sine-Gordon equation under the group of Lie point symmetries are obtained in both differential and difference forms. The invariant discretization of the difference problem corresponding to the boundary value problem for sine-Gordon equation is studied.

Keywords: Lie groups, point symmetries, hyperbolic problems, finite difference schemes.  
2010 Mathematics Subject Classification: 58J70, 58J72, 65N06.

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## 6.6 Computer Science and Technology

Session Organizers: Sahin Uyaver

Computer Science and Technology (CST) is a relatively new paradigm whose applications can be seen in almost all disciplines. CST is one of the sessions of 3rd International Conference of Mathematical Sciences (ICMS 2019), 04-08 September 2019, Maltepe, Istanbul, Turkey organized by Maltepe University. The focus of this session is on the frontier topics in the theoretical and applied engineering and computer science subjects. CST of ICMS 2019 will provide a unique forum to exchange innovative research ideas and recent results while sharing experiences among researchers and technology practitioners. CST is in a broad range of topics related to cryptography, computer vision, new technologies to system designs and modelling/simulation. ICMS conferences have been serving as good platforms for the participants and the society to exchange ideas.

The conference and our session have a stroke balance between theoretical and applied research. The session has been organized with the committees of the conference and it has become an international conference session with a high level of participation from many countries. We have received about 40 papers for our session. With the contents of the papers the session is of pretty innovative and unique ideas and have a potential for the technological appliances.



International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## A New Pseudo Random Bits Generator via 2D chaotic System, Diffusion, and Permeation

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Recently a remarkable correlation between chaos and cryptography has been established. Due to this close correlation, numerous chaos based cryptographic systems were developed during the past years. Some of these cryptosystems are based on the 1D chaotic maps and the others are based on the 2D chaotic maps. The former has limited efficiency and security due to their limited space of the secret key; consequently the regions that exhibit chaotic behaviors are also limited. This limitation problem is a significant disadvantage in the security field. Therefore the current study proposed a new pseudo random bit generator (BRBG) based on new 2D chaotic logistic maps that run side-by-side and start from random independent initial states. The structure of the proposed model consists of three components: firstly, a mouse device as the non-deterministic input sources, secondly, the proposed 2D chaotic system, and thirdly, an initial permutation (IP) table. The statistical tests of the generated sequences of bits are investigated by applying of NIST, which is the most strict statistical test utilized for specifying of purely random sequences.

Keywords: chaotic system, BRBG, permantation table.

2010 Mathematics Subject Classification: 65P20, 74H65, 34C28.

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## Diffuse Representation of Image and Its Applications to Cryptography and Compression

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In this paper, we present a new basis for reducing the amount of data needed to represent the image, then we apply this representation for the cryptography of a database of binary images, next we use this algorithm and the bit-plane decomposition to generalize image encryption for a gray-scale image, where we can restore the original image without loss of information, we also show the use of this representation for lossy compression

**Keywords:** Cryptography, Diffuse representation, Lossy compression.  
**2010 Mathematics Subject Classification:** 68P25, 94A08, 94A60.

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## Comparison of C, Java, Ruby and Matlab Programming Languages Using Fibonacci Algorithm

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**Abstract.** Many programming languages evolved with the development of technology. However, it is still not clear which programming language should be used for which applications since there are not many comparisons of these languages. The aim of this study is to compare the performances of the most frequently used programming languages; C, Java, Ruby and Matlab using Fibonacci algorithm. Fibonacci is chosen for this study because it is a recursive algorithm and is widely used in computer science. Performances of these languages are measured according to the code length, programming effort, runtime efficiency and reliability. The results obtained as a result of these analysis are also shown in this study.

**Keywords:** Fibonacci, Programming languages, C, Java, Matlab, Ruby

**2010 Mathematics Subject Classification:** 11B39, 68Q25, 68W40

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## Image encryption based on highly sensitive chaotic system

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In the recent past, various algorithms for image encryption is introduced based on chaotic systems, some of them appeared weak in some points such as, resisting the chosen plaintext attack and the chosen ciphertext attack with low sensitivity to change of plain images. To overcome such weaknesses, this article proposes a new two-dimensional (2D) hyperchaotic system, which is derived from three maps namely; Henon, logistic, iterative chaotic map with infinite collapse (ICMIC) maps. The performance analysis of this system shows high sensitivity to the initial condition, complicated dynamics, hyperchaotic properties, and better ergodicity. The proposed system is adopted in the design of a new image encryption algorithm. The encryption process is mainly depending on changing of eigenvalues and eigenvectors of original image utilizing a function to change the intensity of pixels, the plain-image is divided into blocks to generate four different keys (multi-key) based on the proposed hyperchaotic system to increase the complexity and reduce the computation time. The partitioned process is repeated several times for each block before obtaining an encrypted image, where the new block in each iteration is multiplied with the four different generated keys. Eventually, to show the efficiency of the encryption image, some performance analysis tests are performed such as; image histogram, NPCR, UACI and pixels correlation.

Keywords: Chaotic system, Hyperchaotic, Encryption, Security.

2010 Mathematics Subject Classification: 65P20, 74H65, 34C28.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Review of the Most Popular Data Science Programs Used Today: Python and R

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As a result of the developments in information technologies, data production is increasing exponentially. This increase in data brings with it the need to produce meaningful information from these data. Many powerful data analysis application software is being developed to meet these requirements of data science. Some of these programs are very popular nowadays. According to many research data in this field, the most popular programming tools in data science are Python, R and SQL. In this study, most commonly used Python and R programming languages in data science are examined. Relevant programming languages, their main features and their advantages compared to each other will be shared comparatively. Thus, it aims to provide comparative information about these popular programs to researchers, students and all parties that will need in this field.

Keywords: Big Data, Data Science, Data Analysis, Data Science Tool, Program, Python, R.  
2010 Mathematics Subject Classification: 68N15, 68N19, 68N99.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## A Study on the Industrial Usage Areas of Modern Information Technologies

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Technological developments have always been a driving force in the development of the industry. In parallel with the developments in information technologies, industrial transformation and development has accelerated. With the introduction of the Internet, this speed has increased much more. Thanks to advances in today's modern information technology, Industry 4.0 transformation is no longer a dream. This transformation; Internet of things, cyber physical systems, augmented reality, artificial and vertical integration, artificial intelligence, learning robots, big data, cloud computing, virtual reality and cyber security. These technologies enable the physical and systemic integration of all related processes of the industrial sector such as management, strategy, planning, production, service, transportation, marketing, sales. Flexible production capacities, speed and efficiency of companies are increasing. In this way, the company paves the way to gain superiority in the sectoral and global competition by working in a more qualified, productive, profitable and agile structure.

This study has been carried out on the usage areas of modern information technologies in industry and their positive contributions. It is aimed to raise awareness in order to make more use of the current information technologies and their contributions in today's journey of Industry 4.0.

Keywords: IT, Digitization, IoT, Industry 4.0, Transformation  
2010 Mathematics Subject Classification: 68M11, 68T45, 68U35.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Market Basket Analysis Developed On The Refrigerator Control System With Machine Learning Algorithms In Data Mining Applications

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The purpose of this study was to analyze the basket basket developed by using machine mining algorithms by using data mining data. Another objective of this study is to find out the critical level in the system by developing a refrigerator control system software application and algorithm using forecasting modeling and machine learning method; and the estimated frequency of consumption. An internet-based simulated application software, written in Visual Studio C#, was developed to interpret data and display the results on the web. The rules of association have been applied to the application software and the data in the database. The data in the refrigerator can be accessed from any location by mobile phone and the current situation can be learned. In this study, it is aimed to develop a model from the data whose results are known with the application developed with the predictive model of data mining models and to estimate the result values for the data sets whose results are unknown. In market basket analysis and association rule; It is aimed to determine the buying habits of customers by finding association-relationship ties between the products that customers purchase during shopping.

**Keywords:** Machine learning, association rules, market basket analysis, data mining, refrigerator control system.  
**2010 Mathematics Subject Classification:** 68RQ99, 68R01, 65C99, 94C99, 68T01.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Sentiment Analysis of Turkish Twitter Data

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In this paper, we present a mechanism to predict the sentiment on Turkish tweets by adopting two methods based on polarity lexicon (PL) and artificial intelligence (AI). The method of PL introduces a dictionary of words and matches the words to those in the harvested tweets. The tweets are then classified to be either positive, negative, or neutral based on the result found after matching them to the words in the dictionary. The method of AI uses support vector machine (SVM) and random forest (RF) classifiers to classify the tweets as either positive, negative or neutral. Experimental results show that SVM performs better on stemmed data by achieving an accuracy of 76%, whereas RF performs better on raw data with an accuracy of 88%. The performance of PL method increases continuously from 45% to 57% as data are being modified from a raw data to a stemmed data.

**Keywords:** Artificial Intelligence, Classifier, Machine Learning, Sentiment Analysis, Turkish, Twitter.



## Motion Estimation from Noisy Image Sequences using New Frequency Weighting Functions

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Motion estimation is a signal-matching technique. It is a key component of target tracking, medical imaging, video compression, and many other systems. This paper presents a four new estimators for frame-to-frame image motion estimation. The estimators of interest are the ROTH impulse response, the smoothed coherence transform (SCOT), the maximum likelihood (ML) and the Wiener estimators. These are all referred to as Generalized Cross-Correlation (GCC)-estimators. These estimators are based on the cross-correlation of the received images and various weighting functions are used to prefilter the received images before cross-correlation. As the performances of the GCC-estimators are considerably degraded by the signal-to-noise ratio (SNR) level, this factor has been taken as a prime factor in benchmarking the different GCC-estimators. For robust motion estimation it has been found that the GCC-Wiener is particularly suited to this purpose. The accuracy of the estimators is also discussed.

Keywords: Motion estimation, Whitening function, Noisy image sequences, GCC-estimators.

2010 Mathematics Subject Classification: 68U10, 62H35, 94A08.

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## A Modification of Gravitational Search Algorithm with Hyper-Ellipsoids

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Gravitational Search Algorithm (GSA) is one of the popular nature inspired metaheuristic method, using the theory of Newtonian's law of gravity and motion in physics. Till now, many variants of GSA have been presented in order to solve different kinds of optimization problems. In the current work, we developed original GSA algorithm using dynamically generated swarm size with mutation operator. For this aim, we generate population inside of and outside of hyper-ellipsoids. Thus, the exploitation and exploration ability of GSA has been improved. We test the performance of proposed approach using some popular benchmark functions including both of low and high dimensional cases. We get the results of proposed approach and original GSA and compare them. According to results, we could say the proposed approach is better alternative for original GSA. Moreover, other population based methods can be improved with this way.

**Keywords:** Metaheuristic, swarm intelligence, gravitational search algorithm, global optimization, hyper-ellipsoid.

**2010 Mathematics Subject Classification:** 49Q99, 68U99, 65K10.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Controlling Electrical Appliance by Thinking in Mind

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Brain Computer Interface (BCI) technologies open up a world of possibilities. They use signals recorded from the brain (e.g., EEG: Electroencephalography) to apply miscellaneous controls and communications without using any external devices or muscle intervention. Their applications include but not limited to: (i) Brain to device control, (ii) Device to brain control, (iii) Brain to Internet communications with an infinite amount of information storage and retrieval, (iv) Mind to mind communication, (v) Memories and feelings transformation, and (vi) Brain to brain control. However, BCI technologies are still in its emerging stages. This paper demonstrates a brain to device control application for controlling electrical appliances by deeming mind thinking signal of the EEG.

Keywords: Appliance, Ardiuno, Brain Computer Interface, EEG, Light Bulb, Neurons.

## Development of an Information System for Storing Digitized Works of the Almaty Akademgorodok Research Institutes

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For decades, scientists of Almaty Akademgorodok research institutes have been conducting enormous research in leading areas of the agro-industrial area producing hundreds of thousands of articles and other documents. However, it should be recognized that the results of these studies remain inaccessible to the vast majority of researchers in the age of the information explosion. One of the reasons for this problem is the lack of a publicly available information repository in the field of agriculture.

For this reason, an integrated distributed information system of Akademgorodok has been created the main objective of which is not only to preserve the rich heritage of the research institutes, but also to provide access to them and the ability to quickly search for the necessary information.

The presented talk describes the architecture of the integrated distributed information system created for storing digitized works of employees of Almaty Akademgorodok research institutes and providing access to them using Web technology. Comparative analysis data storage systems and underlying software for storing digitized works is provided.

This research was supported by Science Committee of Ministry of Education and Science of the Republic of Kazakhstan (grant No. AP05131806).

**Keywords:** integrated distributed information system; data storage; Ceph, GlusterFS, digital object repository; DSpace; CRIS; web portal.

**2010 Mathematics Subject Classification:** 68M14.

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## Encryption Algorithms in Blockchain Technology

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In the digital age, intelligent devices produce data intensively. There is a need for protection in the transmission and storage of this data. Blockchain is a record of encrypted data transfers on the Internet. This new technology uses complex mathematical functions. As the blockchain grows, the solution of these operations becomes more difficult. Since it creates strong encryption on digital identity, it has started to be used in internet of objects, big data, artificial intelligence and robot technology. There is no need for a central authority in blockchain technology. Transactions are carried out directly between the buyer and the seller and in a safe manner. The history of all transactions is publicly available on the network. It provides the opportunity to use secure technologies in critical areas such as finance, health, education, insurance, industry, energy and military. Due to the opportunities it provides, different sectors adapt their business practices to the blockchain technology. In this study, encryption algorithms of blockchain encryption technology will be examined.

Keywords: Blockchain, Cyber security, IoT security, Encryption Algorithms, Digital Identity  
2010 Mathematics Subject Classification: 94A60, 68R01, 14G50, 68P25, 68Q99.

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## Darboux integrability and algebraic invariants of an enzymatic diffusion-reaction system

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Two important mechanisms, Darboux integrability and time-reversibility are studied for proving the existence of a center in the system applying the algorithms of computational commutative algebra. The existence of the first integrals of a three-dimensional enzymatic diffusion-reaction system is investigated. The first integrals are constructed by finding the algebraic invariants of a polynomial ordinary differential equations system.

Keywords: Darboux integrability, time-reversibility, algebraic invariant.  
2010 Mathematics Subject Classification: 37G10, 65P30, 13A50.

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## Weak stabilization of a fractional output for a class of semi-linear Dynamical Systems

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In this paper, we study the stabilization of the state fractional spatial derivative, using Riemann Liouville derivative of order  $\alpha \in [0, 1[$  for a class of semi-linear distributed systems. Then, we develop sufficient conditions for the weak stabilization of a fractional output. Finally, we illustrate the obtained results with numerical simulations.

In this work, the state fractional spatial derivative stabilization of order  $\alpha \in [0, 1[$ , for a class of semi-linear distributed systems, is discussed. We explored conditions that characterize the exponential and weak stabilization of the fractional output. Furthermore, we illustrated the effectiveness of the investigated stabilization theorems by numerical simulations.

This work gives an opening to other questions, this is the case of extending these results to distributed nonlinear systems.

**Keywords:** Distributed semi-linear systems, fractional spatial derivative, output stabilization, weak stabilization.

**2010 Mathematics Subject Classification:** code: 93D15.

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## Fusion of Finger-Knuckle-Print and Finger Vein Recognition using Random Forest Tree

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In this paper, we propose an identification system using multimodal fusion of finger-knuckle-print (FKP) and finger vein (FV) network by adopting several techniques in different levels for multimodal fusion. Therefore, we are proposed the Image-Level fusion and Score-Level for the fusion of these two biological traits. Where, are created optimization multimodal system by enhancing this different levels fusion. In data mining, we are based on Discrete cosine Transform (*2D-DCT*) for features extraction, also we are invested to use Random Forest Tree (*RFT*) for classified and test these statistical features. Our proposed system is given best experimental results that reflect the robustness of system.

Keywords: Biometrics, Fusion, FKP, FV, ROI, DWT, 2D-DCT, RFT, Image-Level, Score-Level.

2010 Mathematics Subject Classification: 68M11, 68T45, 68U35.



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## Extracting a New Fractal and Semi-Variance Attributes for Texture Images Categorization

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### Abstract

Texture feature extraction is one of the essential functions in the field of image processing and pattern recognition. There is a very high demand for finding new attributes for this purpose. The fractal dimension is demonstrated to be an excellent parameter to analyze textures at different scales. In this work, we propose new attributes for image categorization by utilizing two components of texture analysis: fractal and semi-variance characteristics. A set of five attributes is used to investigate different texture patterns. Lacunarity and two other attributes, along with fractal dimension, are four candidates for semi-variance estimation used to ensure a better description of the textured appearance. The Simple K-means method was adapted for clustering purposes and revealed from two to ten different clusters. Subsequently, several classification algorithms were used to categorize a new image form the extracted features; those classification algorithms are Nave bays, Decision tree, and Multilayer Perceptron. Ten-fold cross-validation scheme is also used to reduce the variability of the results.

Keywords :Fractal attributes, Semi-variance two, Texture classification.

2010 Mathematics Subject Classification: 28A80,37F05, 81Q35.

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## A Block-Based Image Encryption Scheme Using Cellular Automata With Authentication Capability

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Cellular automata (CA) are discrete dynamical system formed by a finite array of identical objects called cells. Each cell is endowed with a state which changes at every time-step depending on the states of its adjacent cells at previous time-steps. This feature provides confusion and therefore makes CA attractive in cryptography.

This paper presents an authenticated image encryption algorithm based on CA. To accelerate the process, we divide the image into blocks and use a permutation algorithm to apply chaos on the blocks and then use CA-based algorithm to permute the pixels and change their values simultaneously. The authentication mechanism of our scheme, adjustable to the desired level, can detect slight tampering in the cipher image before full decryption. Furthermore the authentication ability can be adjusted to any desired level. Existing image encryption schemes usually cannot provide parallel processing capability and high sensitivity to changes simultaneously. This study tries to overcome this drawback as well. Our scheme further provides the ability of decryption parts of the cipher image independently from others which is very useful for real time applications. We show that our proposal fulfills desired security properties including large key space, statistical attacks, differential attack and chosen-plaintext attack.

**Keywords:** Image encryption, Cellular automata, chaotic map, Authentication.

**2010 Mathematics Subject Classification:** 68P25 , 94A60, 68P30.

## 6.7 Mathematical Methods in Physics

Session Organizers: Özey Gürtuğ and İzzet Sakallı

The symposium Mathematical Methods in Physics is organized in ICMS 2019, Maltepe University, Istanbul, Turkey, on 04th - 08th September, 2019. The programme of this symposium is mainly oriented towards some recent developments in quantum field theory, gravity and cosmology, nonlinear systems, special functions, boundary problems and some relevant mathematical methods. We hope that all attending this meeting will recall it as a useful and pleasant event. We wish to thank all lecturers and other speakers for their interesting and valuable talks. We also thank all participants for their active participation. And special thanks to our sponsors for their financial supports, which were very significant for realization of this scientific activity.

### **Özey Gürtuğ and İzzet Sakallı**

-Özey Gürtuğ is currently working as a professor at the Faculty of Engineering and Natural Sciences. At the same time he is the Dean of the Faculty of Architecture and Design. He obtained his BSc degree in Electrical and Electronic Engineering at the Eastern Mediterranean University (EMU). He received his MSc and PhD degrees from the Physics Department of EMU. His research interests are gravitational waves, black holes and space-time singularities.

-İzzet Sakallı is currently working as a professor at the Faculty of Arts and Sciences, Physics Department, Eastern Mediterranean University, Famagusta Cyprus. In addition to his academic duties, he also serves as department head. While receiving his BS degree from Osmangazi University, he received his M.Sc. and Ph.D. degrees from Eastern Mediterranean University, under the supervision of Prof. Dr. Mustafa Halilsoy. His research interests are black holes, Hawking radiation, greybody factors, gravitational lensing and wormholes.

## Two Dimensional Free Surface Flows Past an Obstacle

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The free surface problems can be defined as problems whose its mathematical formulation involves surfaces that are to be determined as part of the solution of the problem. This type of problems is characterized by the non-linear condition given by Bernoulli's equation on its free boundary. Thus they are known in scientific literature as problems where the Eulerian description is more practical to model them mathematically. In this work, we try to give a mathematical formulation of two-dimensional free surface flow of inviscid and incompressible fluid considered past an obstacle, using analytical and numerical techniques based on the conformal mapping for reasons of simplification and find some approximate solutions.

**Keywords:** free-surface flow, hodograph transformation, series truncation.

**2010 Mathematics Subject Classification:** 76B10, 76B07, 30C30, 65E05.

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## Analytical Solution for the Conformable Fractional Telegraph Equation by Fourier Method

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In this paper, the Fourier method is effectively implemented for solving a conformable fractional telegraph equation. We discuss and derive the analytical solution of the conformable fractional telegraph equation with three kinds of nonhomogeneous boundary conditions, namely, Dirichlet, Neumann and Robin boundary conditions.

**Keywords:** Conformable fractional telegraph equation, Fourier method, Laplace transform.  
**2010 Mathematics Subject Classification:** 35R11, 42B05, 44A10.

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## Small Divisors in the Solar System

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Consider the Sun, Mercury, Venus, Earth+Moon, Mars, Jupiter, Saturn, Uranus and Neptune as point masses, moving according Newton's inverse-square law of gravitation. Poincaré proved [1] that the general solution of the  $N$ -body problem can be expressed as a formal Fourier series. But Poincaré also proved that such a series generally diverges.

This happens because some small divisors appear when we apply the method of variation of arbitrary constants [2, 3]. A divisor which may vanish during the next  $10^{10}$  years we define as a small divisor. For each pair of planets, there always exists a principal small divisor which could spoil the stability of their orbits after certain time.

Finally we estimate the impact of the small divisors to conclude that they would not destroy the stability of the solar system at least for the next  $10^{10}$  years.

Keywords: Small divisors, Perturbations.

2010 Mathematics Subject Classification: 70F15.

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## Laguerre Polynomial Approach for Solving Functional Differential Equations Involving First Order Nonlinear Delay Terms

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Recently, there exists an increasing interest on models related to delay and nonlinear functional differential equations in many scientific areas such as biology, chemical, physics and engineering. Moreover, the numerical methods for these problems have been developed by many authors. In this study, we consider some high-order delay functional differential equations with variable coefficients and variable delays, which contain first order nonlinear delay terms; then we develop a compatible matrix-collocation method depends on Laguerre polynomials to find the numerical solutions of these type equations subject to the mixed conditions. Additionally, numerical examples and different error analysis techniques are achieved to illustrate the efficiency, usability of our method.

Keywords: Collocation methods , Laguerre polynomials, delay functional differential equations.  
2010 Mathematics Subject Classification: 65L60, 33C45, 34K06.

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## Laguerre Matrix-Collocation Technique to Solve Systems of Functional Differential Equations with Variable Delays

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In this study, a matrix method which depends on collocation points and Laguerre polynomials is developed for solving the approximate solutions of systems of high-order delay differential equations involving variable coefficients and variable delays. These kinds of systems characterized by the present functional delays and which explain many different phenomena and particularly, arise in studies based on biology, physics, chemistry, electrodynamics, and economy and in industrial applications. The proposed method reduces the solution of the mentioned delay system subject to the initial conditions to the solution of a matrix equation with the unknown Laguerre coefficients. Moreover, the approximate solution is obtained in terms of Laguerre polynomials. Besides, some examples along with different error techniques are performed to illustrate the applicability of the method; the obtained findings are scrutinized and interpreted.

**Keywords:** Collocation methods , Laguerre polynomials, system of functional differential equations.  
**2010 Mathematics Subject Classification:** 65L60, 33C45, 34M03.

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## Secure Optical Communication Based on New 2D-Hyperchaotic Map

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Chaos-based cryptography is considered as a preferred way in transferring and processing of the data through different unsecured channels; one of them is the optical channel. The reason behind this important is because of non-periodic chaotic signals; their behavior is nonlinear in addition to their sensitivity to initial conditions, which grant chaos great importance in measuring the unpredictability and randomness. These properties conduct that; the designed system must possess high complexity. Therefore, designing a non-linear system satisfying the aforementioned properties is of high demand. Derived from the existing 1D-sine map, a new nonlinear 2D-adjusted sine map is intended to generate a hyperchaotic behavior; its dynamical properties are studied in terms of fixed points, Jacobian eigenvalues, trajectory, Lyapunov exponent, and bifurcation diagram. The complexity of the 2D-adjusted sine map is investigated using the Approximate Entropy (AE). The generated chaotic signal is modulated with the message and sent as a binary sequence through an optical channel.

Keywords: hyperchaotic system, chaotic map, lyapunov exponent.

2010 Mathematics Subject Classification: 65P20, 74H65, 34C28.

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## The Explicit Relation Between the DKP Equation and the Klein-Gordon Equation

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Petiau-Duffin-Kemmer (DKP) is a first order relativistic equation to describe spin-0 and spin-1 particles. Many researchers have been interested in the DKP equation. E. Fischbach et al, R.A. Krajcik et al have been interested in the equivalence of the DKP equation with the Klein-Gordon (KG) and the Proca equations. V.Ya. Fainberg et al and J.T. Lunardi have treated the equivalence of DKP and KG equation. In this work, we give an explicit relation between the DKP and the KG equations for both the spin-0 particle in  $(1 + 3)$  dimensions and spin-1 particle in  $(1 + 1)$  dimensions. By the use of the system of the DKP equation, we get another system generated by the KG equation. Using this equivalence, we calculate the Volkov-like solution of the DKP equation for the spin-0 particle in the field of an electromagnetic plane wave.

Keywords: Equation of Duffin-Kemmer-Petiau, Algebra DKP, Klein-Gordon equation, Volkov solution.

2010 Mathematics Subject Classification: 35Q40, 35Q60, 35Q70, 70G70, 81T10, 83C50.

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## Soliton Solutions of Gursej Model with Bichromatic Force

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Gursej proposed a spinor field equation which is similar to Heisenbergs nonlinear generalization of Diracs equation. This equation is the first nonlinear conformal invariant wave equation [1]. Recently, many studies have been done on Gursej model to understand the quantum properties and dynamics. [2-5]. Also it is known that, solitons are the solutions of nonlinear wave equations and a special kind of localized waves with particle-like behaviours [6]. Soliton type solutions of Gursej model have been found by the use of Soler ansatz [7,8]. In this paper, we investigate the soliton solutions in Gursej wave equation held in a tilted bichromatic force by constructing their Poincar sections in phase space depending on the system parameters.

Keywords: soliton, spinor, nonlinear dynamic.

2010 Mathematics Subject Classification: 35C08, 35Q51, 81R25, 70K05, 70K43.

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## MHD Micropolar Blood Flow Model through a Multiple Stenosed Artery

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Unsteady blood flow characterized by micropolar fluid in a stenosed artery under the influence of a uniform magnetic field is considered. The governing equations in the cylindrical coordinate system are discretized and solved using the Marker and Cell method, where the pressure is calculated iteratively using the Successive-Over-Relaxation method. The outcomes are tested for numerical stability with desired degree of accuracy. The results carried out include the flow-field profiles of blood, the wall shear stress and the streamlines. The results also predict the pressure drop across the multiple stenoses and show that the pressure drop get the highest value when it cross both stenoses. Blood flow contours have been plotted to understand the flow pattern in the diseased artery, which alters significantly in the downstream of the stenosis in the presence of magnetic field.

**Keywords:** Magnetohydrodynamics, numerical modelling, pressure-correction

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## Total Reduction of Chiral Oscillator and Its Dirac Analysis

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In this study we present the Hamiltonian formulation of dynamical systems produced by second order degenerate Chiral Lagrangian. Chiral Lagrangian is reduced into first order Lagrangian and then for this reduced Lagrangian Dirac- Bergmann algorithm is employed.

Keywords: Second order degenerate Lagrangians, the Dirac-Bergmann algorithm, Chiral Oscillator.  
 2010 Mathematics Subject Classification: 70H45, 70H50, 70H05, 83E05.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Thin-Shell Wormhole in $f(R)$ Gravity

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We present thin-shell wormhole in  $f(R) = R + R_0 + 2\alpha\sqrt{R + R_1}$  gravity coupled with a model of non-linear electrodynamics Lagrangian. We also study the dynamic stability and some physical properties of the thin-shell wormhole.

Keywords: Thin-shell wormhole, Nonlinear electrodynamics,  $f(R)$  gravity.  
2010 Mathematics Subject Classification: 83C15, 83C50, 83C99.

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## Mathematical Beauty in Black Hole Radiation

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Hawking considered how quantum particles behaved close to a black hole, predicting that a black hole irradiated a form of thermal radiation, later termed Hawking radiation [1]. Although the classical black holes are asymptotically flat, especially in the presence of axion and dilaton fields, which are the dark matter and energy candidates, black holes can no longer be an asymptotically flat. The best example for this type is the rotating linear dilaton black hole (RLDBH) [2, 3]. In this study, we revisit the HR radiation problem of the RLDBH [4]. For this purpose, we consider the most advanced scalar perturbations: charged massless spin-0 fields. After separating the covariant Klein-Gordon equation into radial and angular equations, the analytical solutions of those equations are obtained in terms of the confluent Heun functions. Various physical problems are discussed with the obtained analytical solutions: resonance frequencies, quantization and greybody factor [5]. Moreover, we derive the Hawking temperature of the RLDBH by using the Damour-Ruffini-Sannan method. The mathematical beauty of black hole radiation is remarkable during all these processes.

**Keywords:** Hawking Radiation, Black Hole, Dilaton, Axion, Quantization, Greybody, Heun Functions.

**2010 Mathematics Subject Classification:** 35Q75, 37N20, 58D30, 83A05.

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## Solving Advection Equation Using the Natural Decomposition Method

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In this paper, the natural decomposition method (NDM) is used to find an exact solution of non-linear homogeneous and nonhomogeneous advection equation. The Adomian decomposition method is the basis of natural decomposition method. The theoretical analysis of the natural decomposition method is investigated for some equation and calculated with easily computation terms. The results are compared with other method. The evident has illustrated that this method is easy and efficient.

**Keywords:** Natural decomposition method, Adomian decomposition method, Adomian polynomial, Advection equation.

**2010 Mathematics Subject Classification:** 49M27, 35L05, 35C10.

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04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Application of Differential Transform Method and Adomian Decomposition Method for Solving of one Nonlinear Boundary-Value-Transmission Problem

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In this study, we will find the approximate solution of one nonlinear boundary-value-transition problem by using Adomian Decomposition Method and Differential Transform Method. Namely we investigate the nonlinear differential equation,  $y''(x) + y^2(x) = \lambda y(x)$ ,  $x \in [1, 2) \cup (2, 3]$  subject to boundary conditions  $y(1) = y(3) = 0$  and additional transmission conditions at the interior singular point  $x = 2$ , given by  $y(2 - 0) = \gamma_1 y(2 + 0)$ ,  $y'(2 - 0) = \gamma_2 y'(2 + 0)$ . We obtain that using both Adomian Decomposition Method and Differential Transform Method, it is possible to express analytic solutions of nonlinear boundary-value-transmission problem in terms of series without linearization, discretization or perturbation techniques.

Keywords: Adomian Decomposition Method, Differential Transform Method, approximate solution.  
2010 Mathematics Subject Classification: 34B15, 74G15.

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## Mathematical Behaviour of Solutions of the Kirchhoff Type Equation with Logarithmic Nonlinearity

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In this presentation, we consider Kirchhoff type wave equation with logarithmic nonlinearity. We use Galerkin method and potential depth method to obtain the global existence of solutions. The logarithmic nonlinear source term is related with many branches of physics. Cause of this is interest in it occurs naturally in inflation cosmology and super-symmetric field theories, quantum mechanics, nuclear physics [1,2,3,4].

**Keywords:** Existence, Logarithmic nonlinearity.

**2010 Mathematics Subject Classification:** 35A01, 35B40.

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## The Effect of Social Media for a Zika Virus Transmission with Beddington DeAngelis Incidence Rate: Modeling and Analysis

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This paper discuss a modified mathematical model of Zika virus transmission and analyzes the impact of the awareness programs on social media the modification of of Zika Virus model with saturated incidence rate. The Beddington-De Angelis functional responses used to describe the interaction between a suspected human and an infected human. The dynamics of the model were analyzed by identifying the disease-free (DFE) and endemic equilibrium (END). Next Generation Matrix (NGM) was used to determine the Basic Reproduction Number. The stability of DFE and END were analyzed locally by computing the determinant of Jacobian. The DFE was identified as locally stable when the basic reproduction number was less than unity; and was identified as unstable otherwise. Meanwhile, the END was identified as existents when the basic reproduction number was greater than unity. The Routh-Hurwitz Criterion showed that the END was locally stable under a specific condition. A sensitivity analysis was also computed to determine the most influential parameter value of the model. In the end, the stability of DFE and END were also identified numerically depending on certain parameter values.

**Keywords:** Zika Virus, dynamics, social media, reproduction number, Next Generation Matrix, Beddington-De Angelis, saturated incidence rate.

**2010 Mathematics Subject Classification:** 34D20, 65P40, 37M05.

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## Non-Commutative Geometry and Application to Schrödinger Equation with Certain Central Potentials

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We obtain exact solutions of the 2D Schrödinger equation with the central potentials  $V(r) = ar^2 + br^{-2} + cr^{-4}$  and  $V(r) = ar^{-1} + br^{-2}$  in a non-commutative space up to the first order of noncommutativity parameter using the power-series expansion method similar to the 2D Schrödinger equation with the singular even-power and inverse-power potentials respectively in commutative space. We derive the exact non-commutative energy levels and show that the energy is shifted to  $m$  levels, as in the Zeeman effect.

Keywords: non-commutative geometry, solutions of wave equations: bound states, algebraic methods.  
2010 Mathematics Subject Classification: 51P05, 81S10, 14P05.

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## 6.8 Recent Themes on Controllability and Stability of PDE's

Session Organizers: Marcelo M. Cavalcanti and Valéria N. Domingos Cavalcanti

The special session *Recent themes on Controllability and Stability of PDE's* is mainly concerned with stability theory in PDE's, including the natural interactions between control theory and various areas in evolutionary problems.

The topics covered in the session include recent themes in control theory and related topics such as stabilization, exact or approximate controllability, trend to equilibrium, rate of convergence, oscillation of solutions and attractors.

It is our hope that this session will provide a proper setting for specialists to discuss and collaborate on problem of mutual interest.

We are thankful to Professor Hüseyin Çakallı for his kind invitation in organizing this session.

## Controllability Results of Hilfer Fractional Differential Inclusions with Non-Dense Domain in Abstract Space

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The controllability is one of the basic concepts in modern mathematical control theory. In this regard, we will discuss in this paper existence and controllability results of integral solution defined on a real compact interval for fractional differential inclusions in the sense of Hilfer fractional derivative when the linear part is a non-densely defined operator and satisfies the Hille-Yosida condition. These results are investigated with the help of the known facts about fractional calculus, argument of fixed point theorem, concept of multi-valued map and integrated semigroup.

Keywords: Controllability, Semilinear differential inclusions, fixed point theorem, nondense domain.  
2010 Mathematics Subject Classification: 93B05, 34A60, 47H10.

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## A Note on Dirichlet Problem for Partial Differential Equations with Complex Variables in the Bidisc

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In this study the results of the Dirichlet boundary value problem are for homogeneous and inhomogeneous complex partial differential equations are collected and analyzed. This study consists of two chapters. In the first chapter, some basic definitions and theorems from functional analysis and some technical preliminaries are presented. After these chapter 1 is devoted to the investigation of the Dirichlet problem for the one dimensional partial differential equations with complex variable in the unit disc  $\mathbb{D} := \{z : |z| < 1\}$  of the complex plane. In the Chapter 2, I studied the Dirichlet problem for the two dimensional partial differential equations with complex variable in  $\mathbb{D}^2 := \mathbb{D}_1 \times \mathbb{D}_2 = \{z = (z_1, z_2) : |z_k| < 1, k = 1, 2\}$ .

Keywords: Dirichlet problem, complex analysis, model equation, linear equation.  
2010 Mathematics Subject Classification: 97I80, 97I60, 31B10

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## On the Stabilization of Infinite Dimensional Bilinear Systems

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We consider the question of the output stabilization of infinite dimensional bilinear system evolving on a spatial domain  $\Omega$ . Then, we give sufficient conditions for exponential, strong and weak stabilization of the output of such systems. The obtained results are in particular applicable to regional stabilization. Examples illustrate the efficiency of such controls.

**Keywords:** Infinite dimensional systems, bilinear systems, output stabilization, regional stabilization, feedback controls.

**2010 Mathematics Subject Classification:** 93C15, 93C10, 49J20.

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# A Mixed Relaxed-Singular Optimal Controls for Systems of Forward-Backward Stochastic Differential Equations of Mean-Field Type

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In this work, we consider a singular control problem for systems of forward-backward stochastic differential equations of mean-field type (MF-FBSDEs) in which the control variable consists of two components: an absolutely continuous control and a singular one. The coefficients depend on the states of the solution processes as well as their distribution via the expectation of some function of states. Moreover the cost functional is also of mean-field type. We prove in particular, the existence of optimal relaxed-singular controls as well as optimal regular-singular controls for systems of MF-FBSDEs.

Keywords: Mean-field forward backward SDEs, regular-singular control, relaxed-singular control.

2010 Mathematics Subject Classification: 60H10, 60G55, 93E20.

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## Stability Result for an Abstract Delayed Evolution Equation with Arbitrary Decay in Viscoelasticity

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In this paper, we consider the following second-order abstract semilinear evolution equation with past history and time delay

$$u_{tt}(t) + Au(t) - \int_0^{+\infty} h(s)Bu(t-s)ds + Cu_t(t-\tau) = F(u(t)), \quad t \in (0, +\infty),$$

where  $A$ ,  $B$  and  $C$  be a linear operators.  $F$  is a nonlinear function satisfying some conditions. Under suitable conditions on initial data and the kernel memory function  $h$ , we prove the well-posedness by using the semigroup arguments. The stability result is also established defining a suitable Lyapunov functional for a larger class of kernels. Some applications are also given.

Keywords: Abstract evolution equation, past history, time delay.

2010 Mathematics Subject Classification: 35L90, 35B40, 74D05.

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## Hopf-Like Bifurcation and Mixed Mode Oscillation in a Fractional-Order FitzHugh-Nagumo Model

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In this work we investigate the emergence of mixed-mode oscillations and canard explosion, in a planar fractional-order FitzHugh-Nagumo model (FFHN). An algorithm, called Global-Local Canard Explosion Search Algorithm (GLCESA) is developed and used to investigate the existence of canard oscillations in the neighbourhoods of Hopf-like bifurcation points. The appearance of various patterns of solutions is revealed, with an increasing number of small-amplitude oscillations when two key parameters of the FFHN model are varied. The numbers of such oscillations versus the two parameters, respectively, are perfectly fitted using exponential functions. Finally, it is conjectured that chaos could occur in a 2-dimensional fractional-order autonomous dynamical system, with a fractional order close to one. After all, the article demonstrates that the FFHN Model is a very simple 2-dimensional model with an incredible ability to present the complex dynamics of neurons.

Keywords: FitzHugh-Nagumo model, canard explosion, fractional-order system, mixed-mode oscillation, chaos.  
2010 Mathematics Subject Classification: 34D15, 34D20, 34C23.

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## 6.9 Applied Statistics

Session Organizers: Müjgan Tez and Kadri Ulaş Akay

The modern world is full of data on which many important decisions are based. Therefore, accurate analysis and data interpretation capabilities are required almost everywhere. STATISTICS, which manages this process, is a branch of science that manages the process of obtaining usable information starting from collecting raw data. It is the basis of research in all scientific disciplines. There are statistics wherever data is available. Even people who are illiterate use statistics in their daily life without realising it. In general, statistics can be group under two main headings. These are theoretical statistics and applied statistics. The development of methods used in the discovery of the pattern in the data is the work of theoretical statistics. This process involves randomness. The use of these methods on data and inference is a field of applied statistics. Applied statistics is a branch of statistics that is developing with significant momentum from past to present. Applied Statistical Methods cover the basic understanding of statistics necessary to deal with a wide range of practical problems. Especially in recent years with the development of technology and computer science has gained significant importance. Mathematics, health, engineering, decision making, geography, sports, sociology, marketing, government, nursing, planning, banking, economics, research, education, medicine, accounting, business, technology, psychology, business economics, business management, industry, physics, biology, agriculture and industry are used the applied statistics to make both situation analysis and forecasting. This situation shows the importance of statistics.

One objective of this session is to provide an environment in which recent developments in applied statistics are discussed. Another purpose is to contribute to the development of science by targeting scientific interaction among the participants. Besides, it is aimed to prepare a joint working platform with participants from different disciplines.

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## Regularization Schemes for Statistical Inverse Problems

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We study a statistical inverse learning problem, where we observe the noisy image of a quantity through a operator at some random design points. We consider the regularization schemes to reconstruct the estimator of the quantity for the ill-posed inverse problem. We develop a theoretical analysis for the minimizer of the regularization scheme using the ansatz of reproducing kernel Hilbert spaces. We discuss optimal rates of convergence for the proposed scheme, uniformly over classes of admissible solutions, defined through appropriate source conditions.

**Keywords:** Statistical inverse problem, Tikhonov regularization, Reproducing kernel Hilbert space, General source condition, Minimax convergence rates.

**2010 Mathematics Subject Classification:** 65J20, 62G08, 62G20, 65J15, 65J22.

## Forecasting ATM Transactions

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Installing the ATMs to the right and effective points, which are the only channel providing cash transactions among Digital Channels, is the key point in terms of cost decrease and ease of customer access in the banking sector.

The main purpose of this study is supporting installation of a new ATM decision both outdoor and indoor (in the Shopping Centers) by using data mining and forecasting techniques.

We have used 1,115 existing ATM parks for Outdoor ATM transactions forecasting, and 195 points for Shopping Center ATMs Transaction Number Estimation. We have used data mining techniques because of the huge amounts of data. Multivariate regression analysis has been performed according to district / neighborhood population, other bank ATM clustering, number of customers in neighborhood detail, neighborhood saving and neighborhood household income, shopping center size, number of stores and parking capacity data. In this study, R-based R Studio program has been used for all classification and estimation methods.

**Keywords:** Multi-regression, data mining, forecasting, R.

**2010 Mathematics Subject Classification:** 62-07, 62P30.

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## Improved New Liu-Type Estimator for Poisson Regression Models

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Poisson regression models are commonly used in applied sciences such as economics and the social sciences when analyzing the count data. The maximum likelihood method is the well-known estimation technique to estimate the parameters in Poisson regression models. However, when the independent variables are highly intercorrelated, unstable parameter estimates are obtained. Therefore, biased estimators are widely used to alleviate the undesirable effects of these problems. In this study, we proposed a new improved Liu-type estimator as an alternative to other proposed biased estimators. The superiority of the new biased estimator over the existing biased estimators are given under the asymptotic matrix mean square error criterion. Furthermore, Monte Carlo simulation studies are executed to compare the performances of the proposed biased estimators. Finally, the obtained results are illustrated in real data.

**Keywords:** Poisson regression models, Biased Estimators, Multicollinearity.  
**2010 Mathematics Subject Classification:** 62J07; 62J12

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## Generalized First Passage Time Method for the Estimation of the Parameters of the Stochastic Differential Equation of the Black-Scholes Model

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The parameters estimation is one of main problems of the dynamic models in many scientific fields, particularly in economics and finance. Since its appearance in the Black-Scholes formula has become the most used method for valuing options. It has been studied by many authors: Gross , Steele , Lamberton and Lapeyre, Tsay , etc...

In the classical Black-Scholes model some assumptions are required. This paper studies a parameter estimation problem for the Black-Scholes equation through two methods : First Passage Time method (FPT) and Generalized Passage time Method (GPT). We study this problem on a simulated series.

Keywords: Geometric Brownian, Black-Scholes equation, first passage time.  
2010 Mathematics Subject Classification: 60H35, 62P05, 91B70.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Comparison of ARIMA, Holt-Winters, and LSTM Forecasting Models Using Kullback Information Measure

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Various forecasting models such as the Autoregressive Integrated Moving Average (ARIMA) and Holt-Winters that aren't just widely accepted but also exceptionally good predictors of the time series. Recently, Artificial Neural Networks (ANNs) have been widely studied and utilized in the prediction of time series, and their flexible non-linear modeling capacity is the key advantage of deep learning. Long Short Term Memory (LSTM), in particular has been used in the prediction of time-series in financial sector. The objective of this study is to examine and compare different forecasting models in terms of performance on a time series that is considered difficult to predict. This article's core contribution is to contrast the performance of ARIMA, Holt-Winters and a recurrent neural network LSTM with reference to minimization obtained in the Kullback measure of relative information in prediction. The results shows that LSTM network performs well on monthly data from the NIFTY 50 stock index, a real-life time series forecast in comparison with traditional models like ARIMA and Holt-winters.

Keywords: Shannon entropy, kullback measure of relative information, time series forecasting.  
2010 Mathematics Subject Classification: 94A17, 62M10, 62M20.

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## On the Fuzzy Reliability Estimation for Lomax Distribution

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This paper considers two procedures to estimate the fuzzy reliability for Lomax distribution. The first procedure depends on fuzzy reliability definition that uses the composite trapezoidal rule to find the numerical integration, and the second is the Bayesian procedure, which includes different cases depends on sample data and hyper-parameters of a prior gamma distribution with symmetric and asymmetric loss functions.

Keywords: Fuzzy Reliability, Lomax Distribution, Gamma Distribution, Composite Trapezoidal Rule.  
2010 Mathematics Subject Classification: 62-xx, 62F15, 62N05.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## On the Exponentiated Weibull Distribution

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A new family of continuous distributions based on exponentiated Weibull distributions along with its special case named exponentiated Weibull - exponentiated Weibull distribution have been proposed. The probability density, cumulative distribution, reliability, and hazard rate functions are introduced. Moreover, the most essential statistical properties, simulated data, Shannon entropy, relative entropy, as well as the stress strength model are obtained.

Keywords: Exponentiated Weibull Distribution, Shannon Entropy, Relative Entropy, Stress-Strength Model.  
2010 Mathematics Subject Classification: 60E05, 62N05.

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## 2D Markov-Switching Autoregressive (MS AR) Models for Image Segmentation

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2D autoregressive (AR) models have been successfully used in several applications in signal and image processing. See [1], [2] and [3] for their use in image restoration, [6], [7] and [8] for their contribution to textural images analysis and synthesis.

However, 2D AR models do not allow to describe spatial series with fundamental changes and structural breaks in the dynamic of the data. For instance, when the pixels grey levels of a richly textured image are observed the resulting spatial series exhibit an alternance of different spatial dynamics corresponding to texture regimes. To take into account structural breaks occurring across 2D data, we introduce in this work the 2D Markov-switching autoregressive model that allows for the possibility of sudden changes in the values of the parameters of a 2D AR process. This model can capture various key features of image data, such as similar properties of neighboring pixels, a mean level growth of regional volatility or regional asymmetry.

Keywords: Spatial models, Marko Switching, Image segmentation.  
2010 Mathematics Subject Classification: 91B72, 91D35, 91D10.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Entropy as a Measure of Implied Volatility in Options Market

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Volatility estimation is critical for several reasons for stakeholders in stock markets and the concept of Shannon entropy originated in the communication system has been extensively applied in finance. There are several well-known traditional techniques in the literature to measure stock market volatility, in this communication, we focus on comparing two popular techniques, the standard deviation and implied volatility with a methodology based on information entropy. In our study, the empirical analysis is conducted so as to find some relationship between the three different approaches: implied volatility, historical volatility and entropy and all three give a similar kind of sense, maybe not of the same scale but all of them follow the same trend. This paper focuses on the behavior of Indian markets between 2001-2017 for comparative analysis. We have also tried to model implied volatility as a linear combination of historical volatility and entropy and found that the model was heavily dependent on the values of entropy. Calculating implied volatility evolves numerical complexities and replacing it with entropy simplifies the problem. We have used Shannon entropy; using generalised entropies (i.e., entropy with additional parameters) may give better approximations.

**Keywords:** Shannon entropy, stock market volatility, implied volatility.

**2010 Mathematics Subject Classification:** 94A17, 91G99.

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## Using Copulas to Model Dependence Between Crude Oil Prices

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In this study the main endeavor is to model dependence structure between crude oil prices of West Texas Intermediate (WTI) and Brent - Europe by using copulas. The modeling is considered by several Archimedean copulas and also their convex combinations. To estimate copulas parameters, it is relied on nonparametric estimation (Genest et al. [1]) and also maximum likelihood estimation. Also, to select the right copula, nonparametric and semi-parametric procedure (Genest et al. [1, 2]), GOF test (Genest et al. [3]) and also Akaike information criterion (AIC) are used. GOF test results show that by the nonparametric estimation, all of the selected AC and also their convex combinations are not able to model the dependency of crude oil prices. While by the maximum likelihood estimation, Clayton family and also convex combinations of Clayton and several other families are able to model the dependency of the mentioned crude oil prices. By results of calculations it is summarized that convex combination of the selected copulas improves fitness of them to model dependence of the crude oil prices data.

**Keywords:** Akaike information criterion (AIC), Copulas, Goodness of fit test (GOF), Linear convex combination, Parameter estimation.

**2010 Mathematics Subject Classification:** 62H05, 62H12, 62H20.

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## 6.10 Geometry and Mathematical Education

Session Organizer: İlhan Gül

Geometry is proving to be an increasingly powerful tool that improves its ties to other branches of mathematics such as analysis, topology, algebra, and so on, as well as to theoretical physics research.

The *Third International Conference of Mathematical Sciences* (ICMS2019) was held on the campus of Maltepe University, Istanbul, during the period of 4-8 September, 2019. This meeting had brought together mathematicians interested in geometry and its applications. The aim of this conference was giving lectures on new results and exchanging ideas.

I wish to thank to Professor Hüseyin Çakallı for his kind invitation in organizing this session. Also, I would like to sincerely thank to all participants for their contribution to this abstract book.

International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Perfect Fluid Pseudo Concircular Ricci Symmetric Spacetimes with Codazzi Type Z-Tensor

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This paper deals with pseudo concircular Ricci symmetric spacetimes with Codazzi type Z-tensor. Firstly, we obtain several properties of the vector field related by these spacetimes. After that, we consider a dust fluid for these spacetimes and we find some theorems.

**Keywords:** Pseudo concircular Ricci symmetric spacetime, perfect fluid, Z-tensor.

**2010 Mathematics Subject Classification:** 53B15, 53B20, 53C25.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## On Ruled Surfaces of Natural Lift Curves and Tangent Bundle of Pseudo-Sphere

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In this study, we give a one-to-one correspondence between the natural lift curves on tangent bundles of Lorentzian unit sphere  $TS_1^2$  and hyperbolic unit sphere  $TH^2$  and spacelike or timelike ruled surfaces in  $IR_1^3$ . Then we present some propositions for the properties of ruled surfaces generated by natural lift curves. We calculate striction curve, shape operator, Gaussian curvature and mean curvature of the ruled surface generated by the natural lift curve. We present developability condition of these ruled surfaces. Finally, we give an example to support our main results.

**Keywords:** Tangent bundle, ruled surface, unit dual pseudo-sphere.

**2010 Mathematics Subject Classification:** 53A04, 53A25.

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## On Infinitesimal Transformations Of Weyl Manifolds

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A Weyl manifold is a conformal manifold equipped with a torsion free connection preserving the conformal structure. In order to make computations in a conformal gauge invariant way, it is better to work with weighted tensors and Weyl's covariant derivative which will be called prolonged covariant derivative. In this work, by considering the weights of pseudo-quantities, we examine infinitesimal transformations on Weyl manifolds and we obtain some results by using the definition of prolonged(extended) Lie derivative which was first introduced by Germanov [2].

**Keywords:** Infinitesimal transformation, prolonged (extended)Lie derivative.

**2010 Mathematics Subject Classification:** 53B05, 53B20.

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## Minimum distance between two ellipses

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We consider the distance problem between two ellipses in  $\mathbb{R}^3$ . This problem arises in widely disparate fields as celestial mechanics [1], computer animation, computer vision, CAD/CAM [2] and so on. We prove that in the general case, the complex critical points of the square of the distance between two ellipses are at most sixteen and they correspond to the roots of sixteenth degree polynomial which coefficients are real and depend explicitly of the ellipses equations. We prove that the real critical points are between four and sixteen.

We give as example the distance between Neptune and Pluto. Then both ellipses have the Sun as a common focus; the critical points are six: one maximum, three saddle points and two local minima. We prove that the global minimum is about 2.52 astronomical units.

Keywords: Ellipses, Distance, Euclidean space

2010 Mathematics Subject Classification: 51N20.

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International Conference of Mathematical Sciences (ICMS 2019)  
04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Some Properties of Generalized Complex Space Forms

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In the present paper, we determine the holomorphic curvature tensor of generalized complex space forms and study some properties of this tensor in generalized complex space forms. Moreover, we present results on generalized complex space forms satisfying curvature identities named Walker type identities.

**Keywords:** Generalized complex space form, Holomorphic curvature tensor, Walker type identity.

**2010 Mathematics Subject Classification:** 53B20, 53B35, 53C25, 53C55.

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International Conference of Mathematical Sciences (ICMS 2019)  
 04 September - 08 September 2019, Maltepe University, Istanbul, Turkey

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## Pointwise Slant Semi-Riemannian Submersions from Lorentzian Para-Sasakian Manifolds

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The purpose of this paper is to study pointwise slant semi-Riemannian submersions from Lorentzian para-Sasakian manifolds. Several basic results in this point of view are proven in this paper.

**Definition 1.** Let  $F$  be a semi-Riemannian submersion from a Lorentzian (para) contact manifold  $(M, \phi, \xi, \eta, g_M)$  of dimension  $(2m+1)$  onto  $(N, g_N)$  be a Lorentzian manifold of dimension  $n$ . If for each  $p \in M$  the angle  $\theta(X)$  between  $\phi X$  and the space  $(\ker F_*)_p$  is independent of choice of the nonzero vector  $X \in \Gamma((\ker F_*) - \{\xi\})$ , then  $F$  is called a pointwise slant submersion.

We call the angle  $\theta$  a slant function of the pointwise slant submersion on  $M$ .

**Theorem 1.** Let  $F$  be a Lorentzian almost (para) contact manifold  $(M, \phi, \xi, \eta, g_M)$  onto  $(N, g_N)$  be a Lorentzian manifold. Then  $F$  is a pointwise slant submersion if and only if  $\varphi^2 = \cos^2 \theta(\varepsilon I + \eta \otimes \xi)$ .

Keywords: Slant submersion, pointwise slant submersion, Lorentzian para Sasakian manifold.  
 2010 Mathematics Subject Classification: 53C43, 53C50, 53C15.

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## Geometrical view on set-theoretical solutions of Yang-Baxter equation via Lie algebras

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In this present paper, we tackle set-theoretical solutions of Yang-Baxter equation in Lie algebras. In addition to these, we obtain new solutions with the help of defining new operation on this structure. And also, we give a relation to obtain a general set-theoretical solution of Yang-Baxter in this structure. Finally, we put an interpretation on these solutions from the point of geometrical view.

**Keywords:** Yang-Baxter equation, Lie algebras, Set-theoretical solutions  
**2010 Mathematics Subject Classification:** 11F22, 16T25, 17B01.

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## 6.11 Algebra and Number Theory

Session Organizer: Temha Erkoç Yılmaztürk

Algebra and number theory are very important branches of mathematics at the forefront of current mathematical research. They play a significant role in different areas of mathematics from topology to geometry and multidisciplinary studies from chemistry to engineering. This session provides a setting for discussing recent developments in some topics such as algebraic structures, Lie algebras, coding theory. It is a good opportunity for participants coming from many countries.

We would like to thank all participants for their contributions to the Conference program.

## On the Aggregating of Some Fuzzy Relations and their Related Structures

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The main goal of this presentation is to investigate the aggregation of diverse families of binary fuzzy relations, fuzzy filters, and fuzzy lattices. Some links between these families and their images via an aggregation are explored

Keywords: Aggregation, fuzzy relation, fuzzy lattices, fuzzy filter, trace.  
2010 Mathematics Subject Classification: 03E72, 06D72, 06B10, 97H50.

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## An Abstract Characterization of Menger Algebras of Strongly Quasi-Open Multiplace Maps

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Let  $X$  and  $Y$  be topological spaces. A map  $f : X \rightarrow Y$  is quasi-open if  $\text{Int}(f(U)) \neq \emptyset$  for every non-empty open set  $U \subset X$ . We say that a map  $f : X^n \rightarrow X$  is a strongly quasi-open map if for any non-empty set  $V \subset X^n$  for which all projections have non-empty interiors, the interior of  $f(V)$  is non-empty. Let  $Q(X^n, X)$  denote the Menger algebra of strongly quasi-open maps from  $X^n$  to  $X$  with composition of functions:

$$[fg_1 \dots g_n](a_1, \dots, a_n) = f(g_1(a_1, \dots, a_n), \dots, g_n(a_1, \dots, a_n))$$

A topological space  $X$  is said to be a  $T_D$ -space if for every point  $\xi$  in  $X$  the set  $\{\bar{\xi}\} \setminus \{\xi\}$  is closed. Obviously, each  $T_D$ -space is  $T_0$ -space and each  $T_1$ -space is  $T_D$ -space. We call a topological space  $X$  a  $T_D^+$ -space if it is a  $T_D$ -space with no one-point open sets and if for every point  $\xi$  in  $X$  and for every open set  $U$  containing  $\xi$  the set  $U \cap (X \setminus \{\bar{\xi}\})$  is not empty. Let  $X$  be  $T_D^+$ -space that has an open base, each element of which is an image of  $X$  under a quasi-open map and let  $\Lambda$  be a class of all such spaces.

**Theorem 1.** Let  $X, Y \in \Lambda$ . The Menger algebras  $Q(X^n, X)$  and  $Q(Y^n, Y)$  are isomorphic if and only if the topological spaces  $X$  and  $Y$  are homeomorphic.

Keywords: Menger algebra, strongly quasi-open map.

2010 Mathematics Subject Classification: 20N15, 54C10.

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## New Types of Uninorms on Bounded Lattices

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In this paper, we propose two new methods for generating uninorms on bounded lattices under some additional assumptions on its neutral element based on the existence of t-norms and t-conorms on bounded lattices. As a by-product of these methods, the smallest and the greatest uninorms on bounded lattices are obtained. In addition, an illustrative example is given to clearly understand these new types of uninorms on bounded lattices.

**Keywords:** Bounded lattice, Neutral element, T-conorm, T-norm, Uninorm.

**2010 Mathematics Subject Classification:** 03B52, 06B20, 03E72, 94D05, 97E30.

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## Sheffer Stroke Reduction for Some Algebraic Structures

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In this talk, we give Sheffer stroke reduction for some algebraic structures such as MV-algebras, Wajsberg algebras, BCK-algebras and etc. by means of defining all operators in these structures only via Sheffer stroke operator. We also give some equalities and inequalities which are used in these constructions. In addition, we examine whether there is a transition from one of these structures to another by the help of this reduction.

**Keywords:** Reduction, Sheffer stroke operator, MV-algebras, Wajsberg Algebras, BCK-algebras.

**2010 Mathematics Subject Classification:** 06D35, 03F50, 06F99.

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## Some Properties of EM Rings

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A commutative ring  $R$  is called an EM ring if for each polynomial  $f$  in  $R[x]$  there exists  $a \in R$  such that  $f = ag$  and  $g$  is regular. The class of EM-rings includes integral domains, principal ideal rings, and PP-rings, while it is included in Armendariz rings, and rings having a.c. condition. For Noetherian rings, EM rings and generalized morphic rings are equivalent. We investigate when  $R$ , the polynomial ring  $R[x]$  and the amalgamated duplication ring  $R \bowtie I$  are EM rings.

**Keywords:**Polynomial ring; Power series ring; Annihilating content; EM-ring; Generalized morphic ring; Zero-divisor graph. .

**2010 Mathematics Subject Classification:** 13F20, 13F25, 13E05.

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## On a Generalized Identity of a Prime Ring Involving $b$ -Generalized Derivations

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In recent years many effective results regarding semiprime rings obtained by a number of authors. The main purpose when treating an additive map is to describe the form of the map or the structure of the ring.

Recently, [2] Kosan and Lee propose a new definition:

**Definition 1.** Let  $d : R \rightarrow Q$  be an additive map and  $b \in \mathbb{Q}$ . An additive map  $F : R \rightarrow Q$  is called a left  $b$ -generalized derivation with associated mapping  $d$ , if  $F(xy) = F(x)y + bxd(y)$  for all  $x, y \in R$ .

In the light of this definition, the main results obtained by a number of authors is stated.

Let  $R$  be a prime ring and  $L$  be a noncommutative Lie ideal of  $R$ , let  $F$  be a left  $b$ -generalized derivation associated with the map  $d$ . Supposing that  $a$  is a fixed element of  $R$  such that  $aF(x)^n = 0$  for all  $x \in L$  where  $n$  is a fixed positive integer, under some assumptions on the ring, the characterization of the maps is being treated.

Keywords: Prime ring,  $(b-)$ generalized derivation, Martindale quotient ring, generalized polynomial identity.  
 2010 Mathematics Subject Classification: 16N60,16R60,16W25.

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## On cyclic codes of length $8p^s$ over $\mathbb{F}_{p^m} + u\mathbb{F}_{p^m}$

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Prange and Berlekamp [1, 2] first introduced and studied cyclic codes and constacyclic codes respectively. In the theory of error-correcting codes, these codes have nice algebraic structures and can be easily encoded and decoded using linear shift registers, which explains their preferred role from the engineering perspective. Many researchers studied the algebraic structure of linear codes over various finite rings. In this paper, we establish the algebraic structure of all cyclic codes and their duals of length  $8p^s$  over the chain ring  $\mathbb{F}_{p^m} + u\mathbb{F}_{p^m}$  by considering three cases:  $p^m \equiv 1 \pmod{8}$ ,  $p^m \equiv 5 \pmod{8}$  and  $p^m \equiv 3 \pmod{4}$ . We also find out the number of codewords in each of these cyclic codes. Besides this, we list some self-dual cyclic codes of length  $8p^s$  over  $\mathbb{F}_{p^m} + u\mathbb{F}_{p^m}$ . Also we determine  $\mu$ -constacyclic codes length  $8p^s$  over  $\mathbb{F}_{p^m} + u\mathbb{F}_{p^m}$  by establishing a one to-one correspondence between cyclic and  $\mu$ -constacyclic codes.

**Keywords:** Cyclic codes, constacyclic codes, dual codes, self-dual codes.

**2010 Mathematics Subject Classification:** 94B15.

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## On Products of Irreducible Characters

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Let  $G$  be a finite group and let  $\chi$  be a character of  $G$ . It's well known that the product of  $\chi\bar{\chi}$  is also a character of  $G$ , where  $\bar{\chi}$  is the complex conjugate character of  $\chi$ . Thus,  $\chi\bar{\chi}$  may be expressed as an integer linear combination of some irreducible characters of  $G$ . There are some research articles on products of irreducible characters to classify finite solvable groups. For example, Adan-Bante has completely classify solvable groups which have a faithful irreducible character  $\chi$  such that  $\chi\bar{\chi}$  has a unique non-principal irreducible constituent [1]. In this talk, we give some results about the relationship between the structure of a finite solvable group  $G$  and the kernels of irreducible constituents of the character  $\chi\bar{\chi}$  where  $\chi$  is a nonlinear irreducible character of  $G$ .

Keywords: Products of characters, derived length, character degrees.

2010 Mathematics Subject Classification: 20C15.

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